VisWeek 08
VIS • INFOVIS • VAST

Edge Groups: An Approach to Understanding the Mesh Quality of Marching Methods

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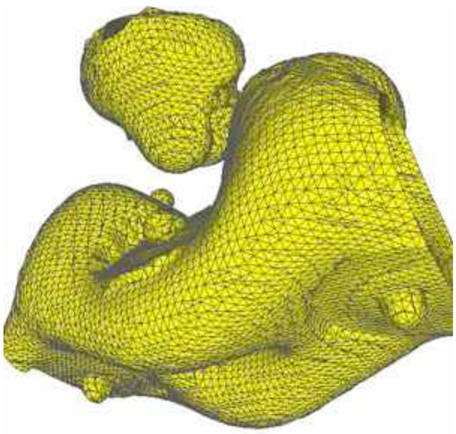
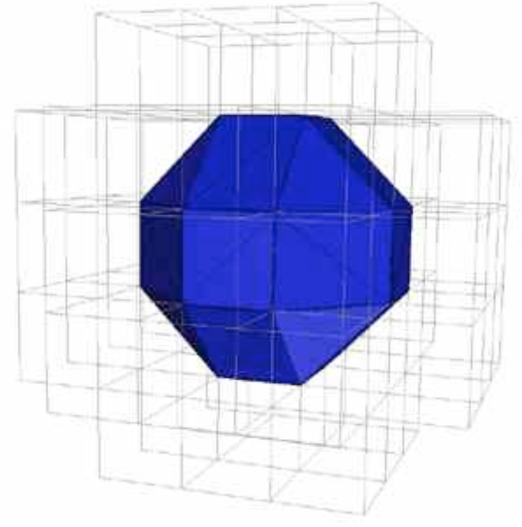
Polygonal Approximation of Isosurfaces (Meshes)

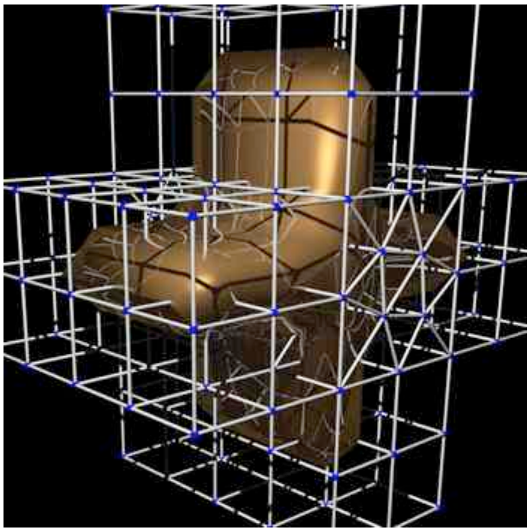
- Possible resulting meshes:
  - Continuous patches
  - Often closed
  - More than one connected component
  - Holes
  - Ambiguous Cases



How to do it ?

Divide-and-Conquer





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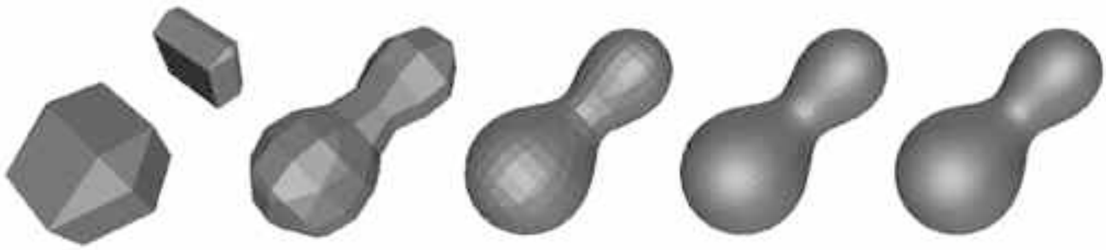
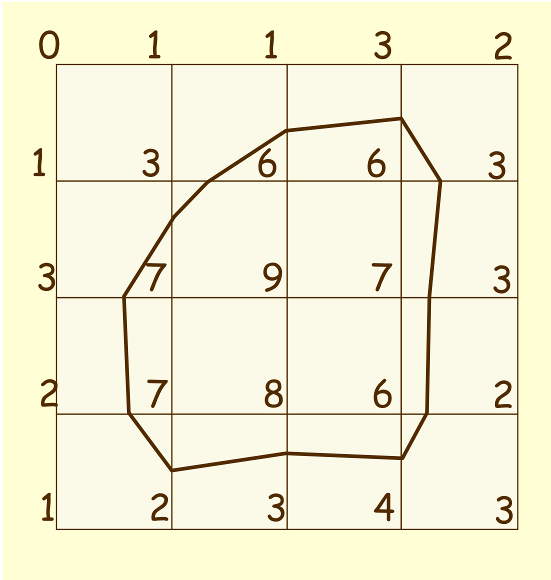


Grid size=10 70 Facets | Grid size=5 220 Facets | Grid size=2 1700 Facets | Grid size=1 6800 Facets | Grid size=0.5 27000 Facets

2D Problem

- Consider 5x5 image, isovalue = 5



2D Problem

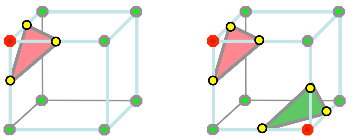
- Consider rectangular cells individually
- Classify each vertex as inside or outside the isosurface (polarity test)
  - How many cases can we have ? How about in 3D ?
- How to compute Intersections ?
- How to maintain continuity ?
- How to implement ? Easy and Fast way ?

Marching Cubes

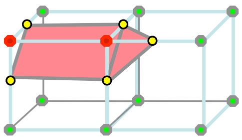
- For each cell:
  - Polarity Test (classify inside/outside each vertex)
  - Create a case index
  - Get list of edges with intersection from table
  - Compute intersections using interpolation

Marching Cubes

- Isosurface extraction from voxels [Lorensen 87]
  Assumes Linear Interpolation between data



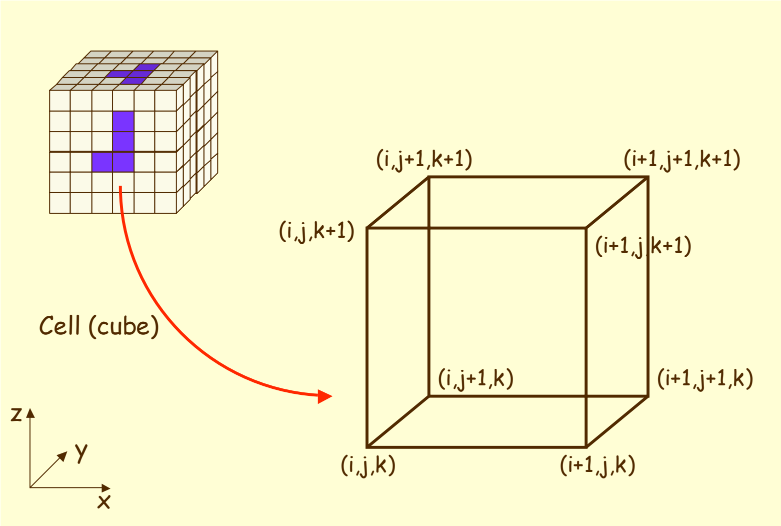
- Mesh extraction
  Connect surface intersections



- Render Mesh using traditional techniques

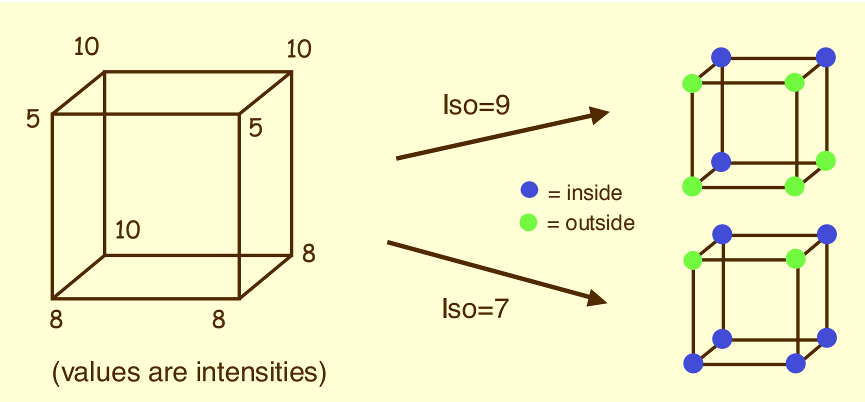
Marching Cubes

- For each voxel:



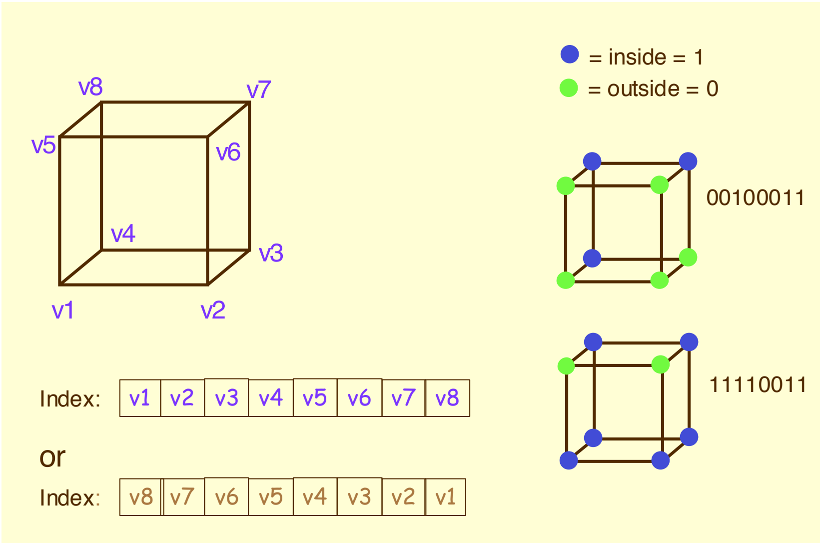
Marching Cubes

- Classify the state of vertices (vertex index)



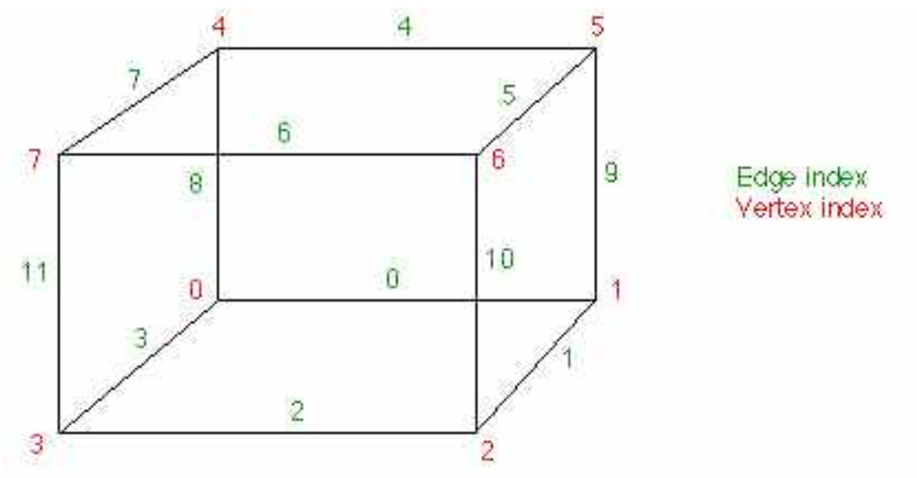
Marching Cubes

- Create Vertex Index



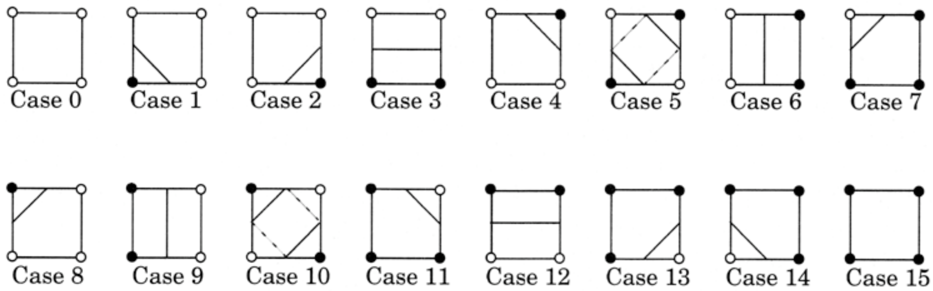
Marching Cubes

- Vertex Indexes and Edge Indexes

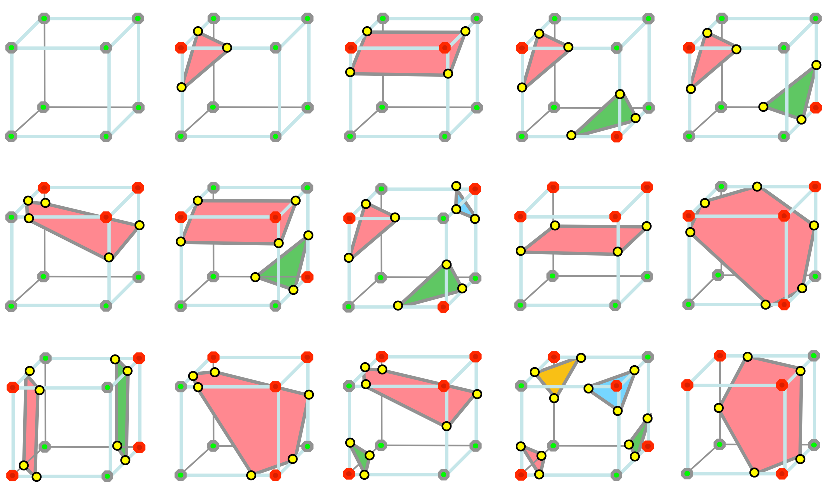


Marching Cubes

- Create Edge List
  - Use an edge index to encode the list of edges cut by the isosurface
  - 2D cell index: 4 bits (2^4 cases)



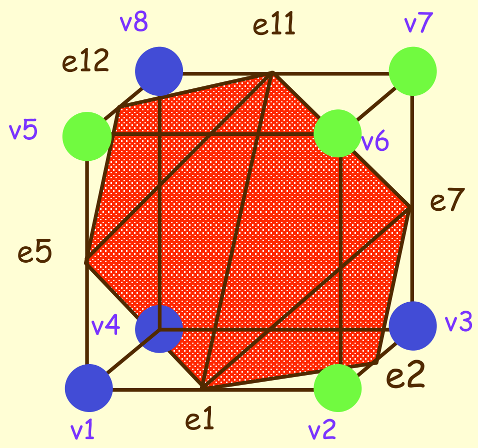
Marching Cubes Cases



Marching Cubes

- Edge list example:

- Index = 10110001
  - triangle 1 = e5, e11, e12
  - triangle 2 = e1, e11, e5
  - triangle 3 = e1, e7, e11
  - triangle 4 = e1, e2, e7



Marching Cubes

- Example:
  - Vertex 0 and 3 are below the isosurface
    - Vertex index =

Marching Cubes

- Example:
  - Vertex 0 and 3 are below the isosurface
    - Vertex index = 0000 1001 = 9

```
cubeindex = 0;
   if (grid.val[0] < isolevel) cubeindex |= 1;
   if (grid.val[1] < isolevel) cubeindex |= 2;
   if (grid.val[2] < isolevel) cubeindex |= 4;
   if (grid.val[3] < isolevel) cubeindex |= 8;
   if (grid.val[4] < isolevel) cubeindex |= 16;
   if (grid.val[5] < isolevel) cubeindex |= 32;
   if (grid.val[6] < isolevel) cubeindex |= 64;
   if (grid.val[7] < isolevel) cubeindex |= 128;
```

Marching Cubes

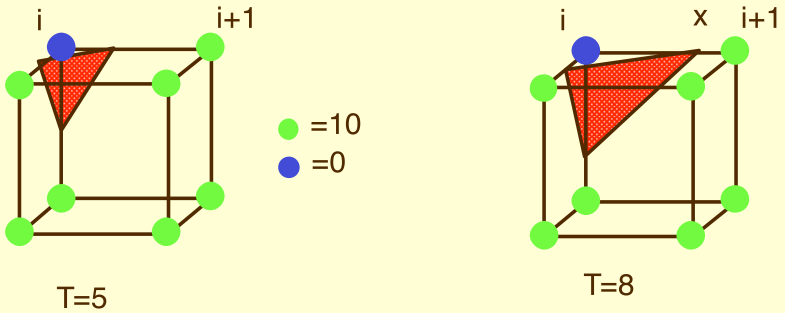
- Example:
  - Vertex 0 and 3 are below the isosurface
    - Vertex index = 0000 1001 = 9
    - EdgeTable[9] = 1001 0000 0101 (905 hex)
      - edges 11, 8, 2 and 0 are cut, compute intersection vertices
    - Triangle table (up to 5 triangles):
      - TriangleTable[9] = {0,11,2,8,11,0,-1,-1,...,-1}

Marching Cubes

- For each triangle, find an vertex location along the edge using linear interpolation of the values at the edge's two end points (*voxels*)

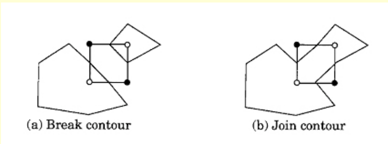
$$x = i + \left(\frac{T - v[i]}{v[i+1] - v[i]} \right)$$

(T is iso-value)

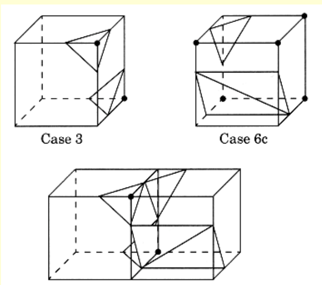


Marching Cubes

- Ambiguous Cases
  - 2D cases
    - Break contour
    - Join contour
  - Both are valid



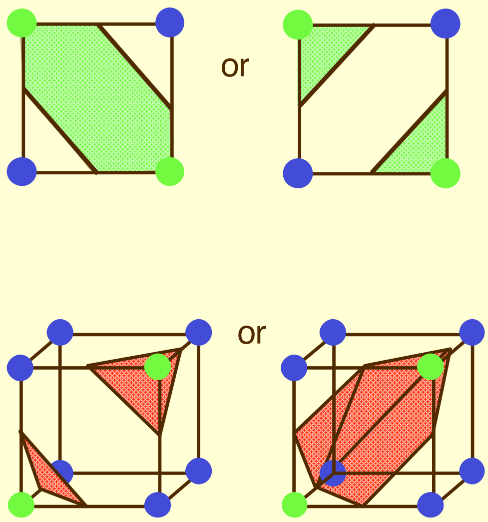
  - 3D cases: may generate a hole (must be eliminated)



Marching Cubes

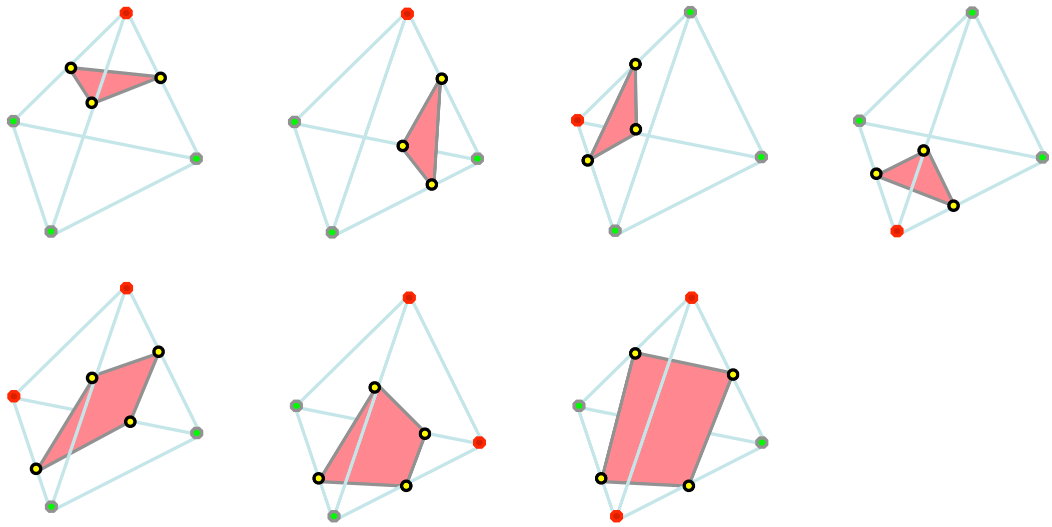
- Ambiguous Cases

  - Ambiguous cases: 3, 6, 7, 10, 12, 13
  - Adjacent vertices: different states
  - Diagonal vertices: same state
  - Need to decide which case is correct



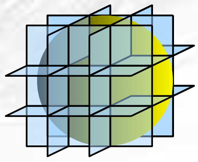
Marching Tetrahedra

- [Doi an Koide 91]



Marching Cubes - (Lorensen/Cline, 1987) 5178 citations 10/29/2008

- Code is simple, efficient and produces stable results
- Table-driven implementation



compute vertex polarity
↓
identify cell case → triangulation table
↓
identify active edges
↓
generate intersection vertices
↓
generate triangulation

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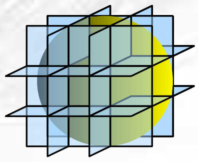

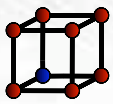

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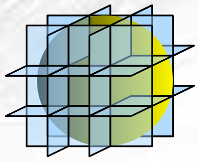

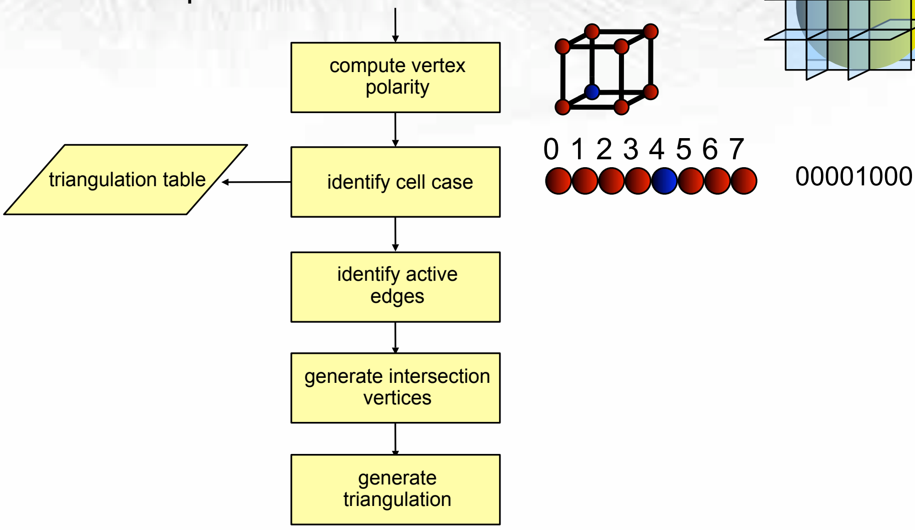

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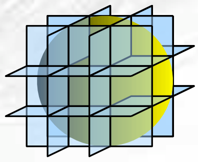

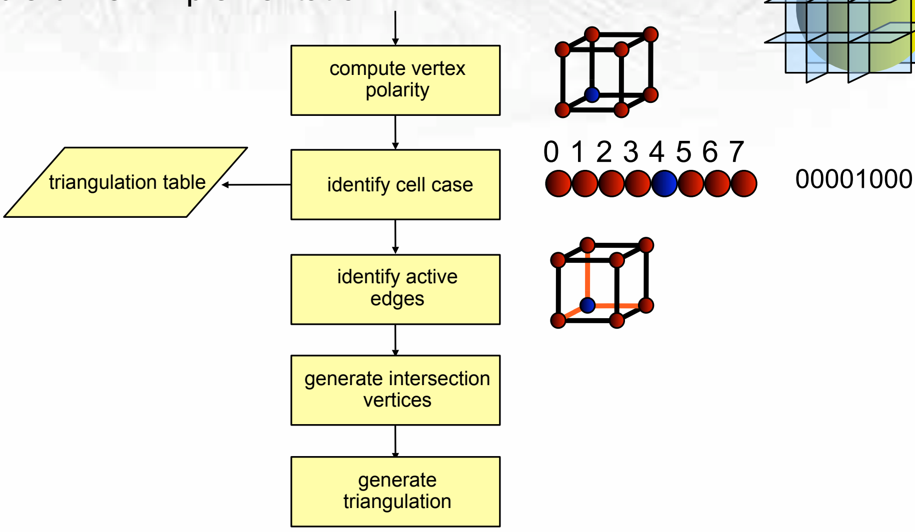

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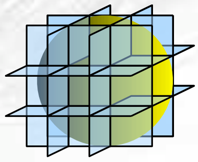

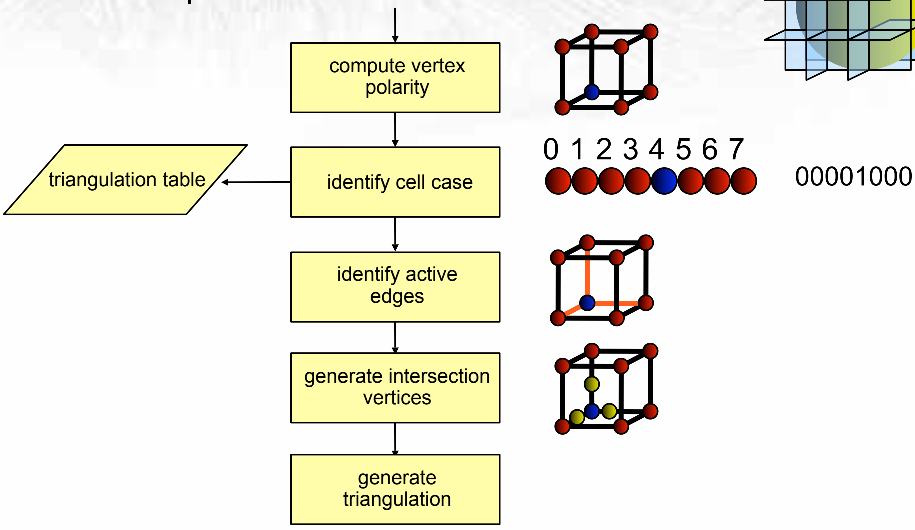

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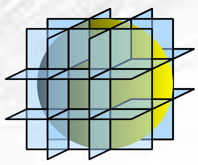
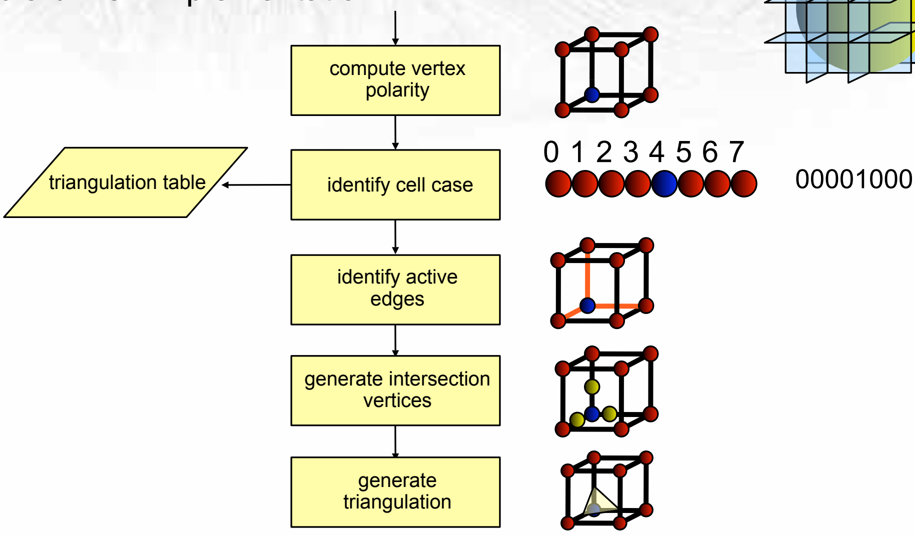
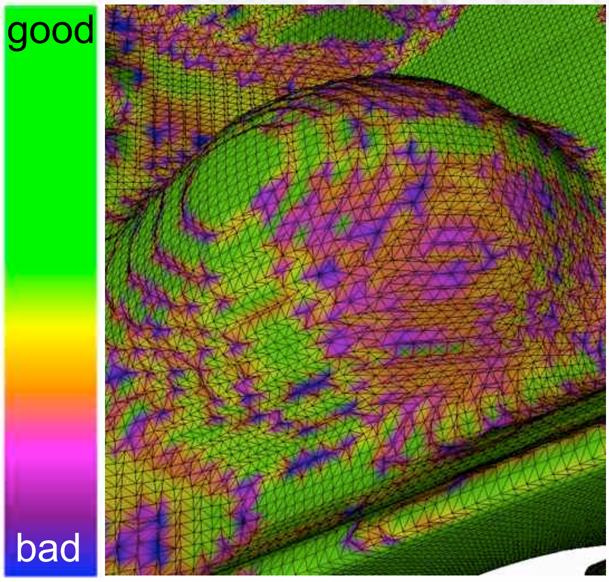
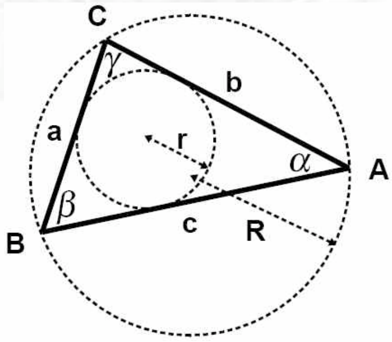




Triangle Quality of Marching Cubes

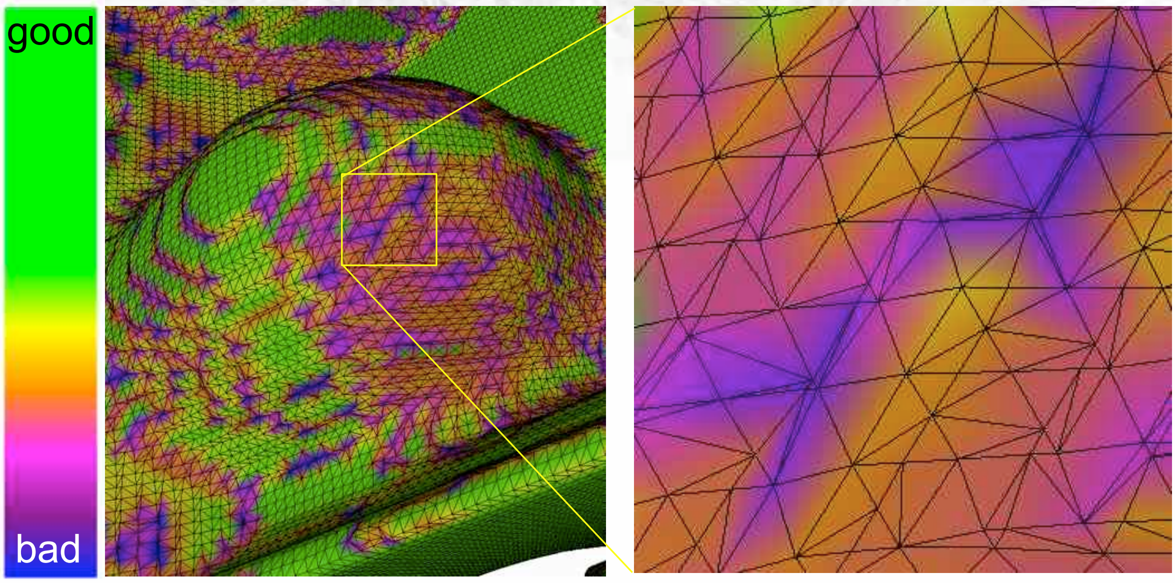




Radii ratio (2r/R)

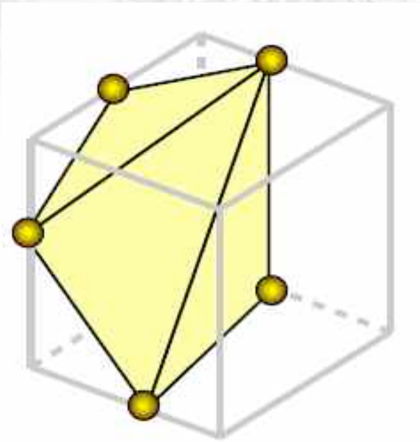
Triangle Quality of Marching Cubes

- Mostly good quality triangles. But some very poor quality triangles



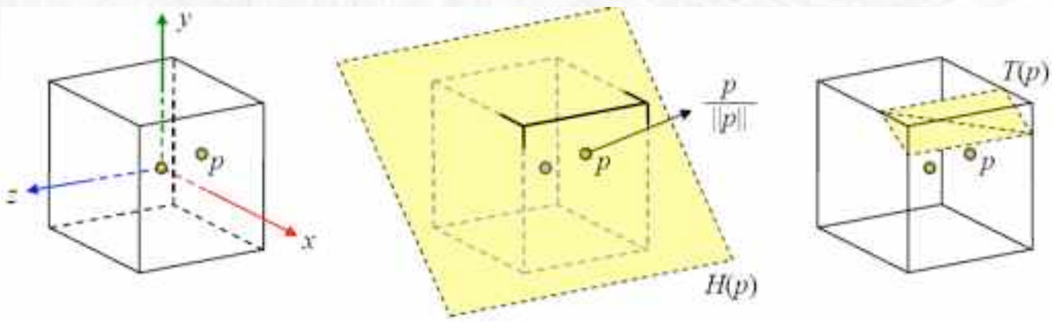
Triangle Quality of Marching Cubes

- In this work we propose a way to understand why this happens.



Mesh quality generated by MC

- Evaluate all possible planar intersections between the isosurface and the cell



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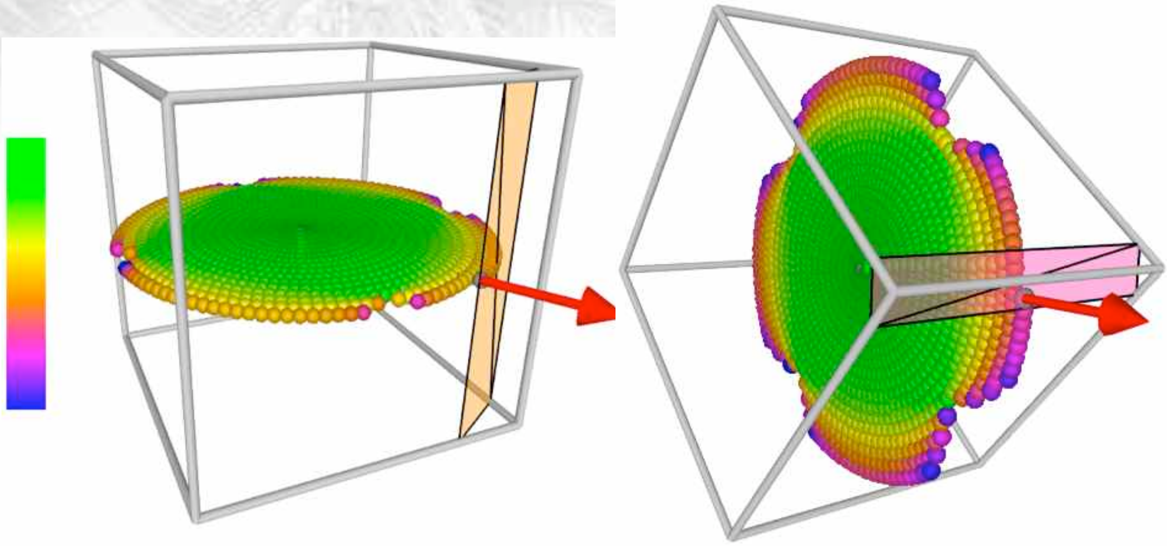
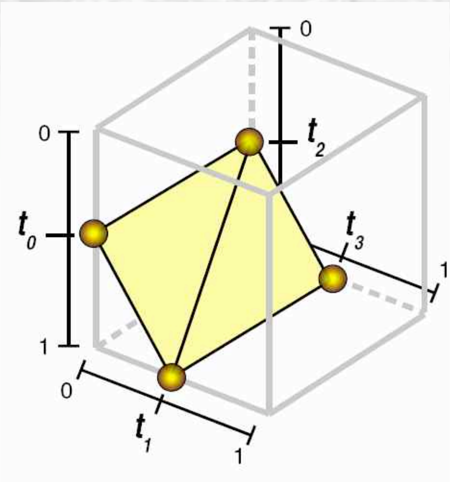


Edge Transformations for Improving Mesh Quality of Marching Cubes

Mesh quality generated by MC



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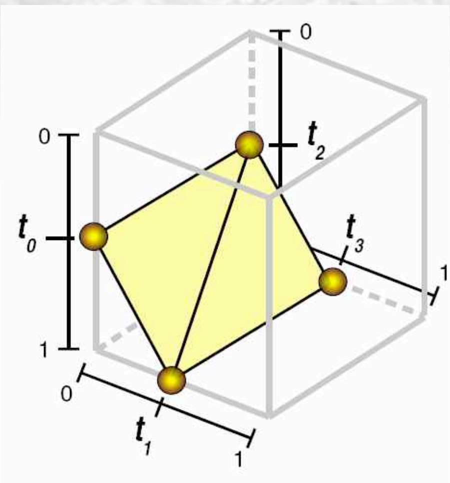
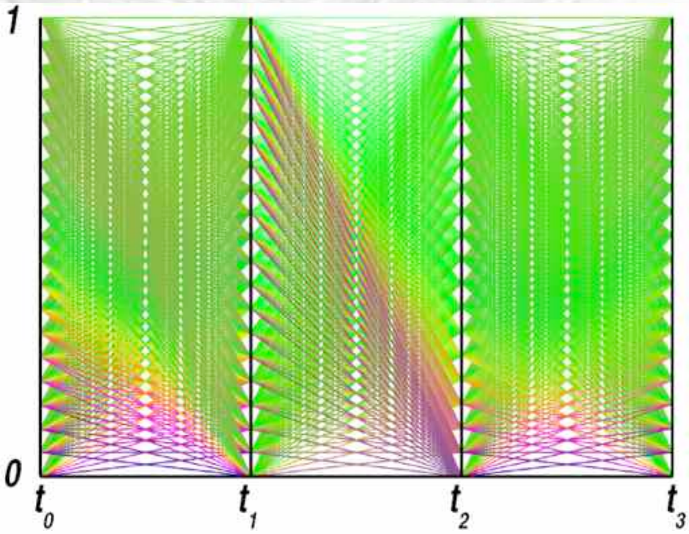
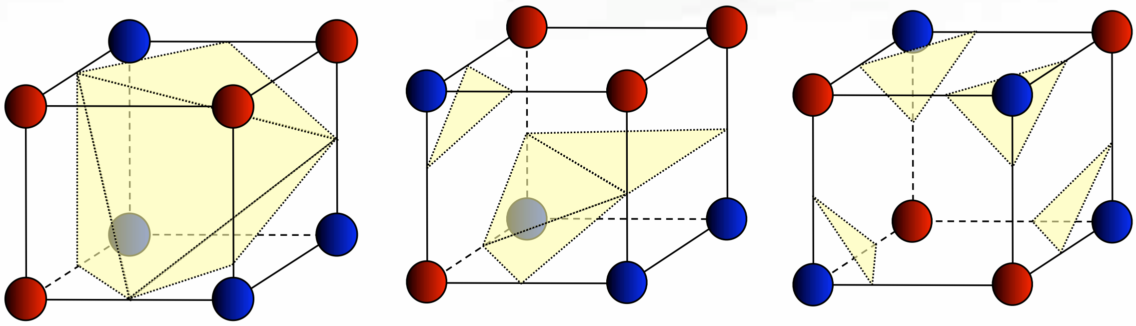




Mesh quality generated by MC

- Difficult analysis of parameters



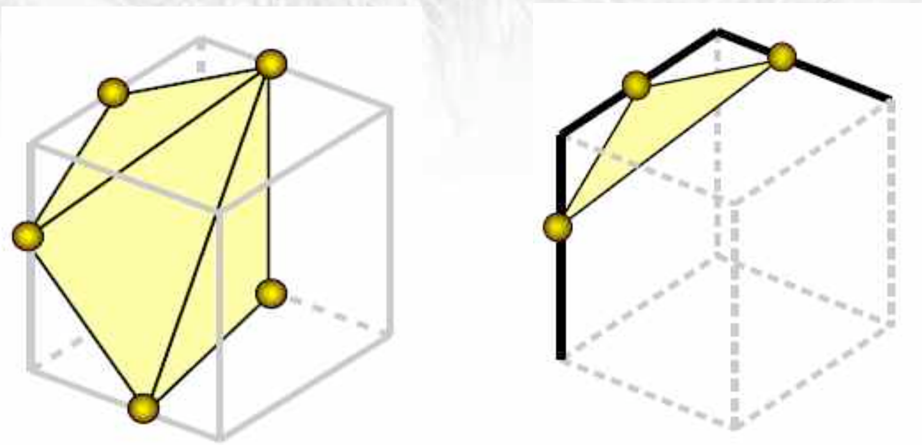
6 parameters

8 parameters

12 parameters

Triangle Quality of Marching Cubes

- Look at each triangle individually



Important Observation:

Each triangle generated by MC has vertices only over active edges. Therefore, it does not generate arbitrary triangles

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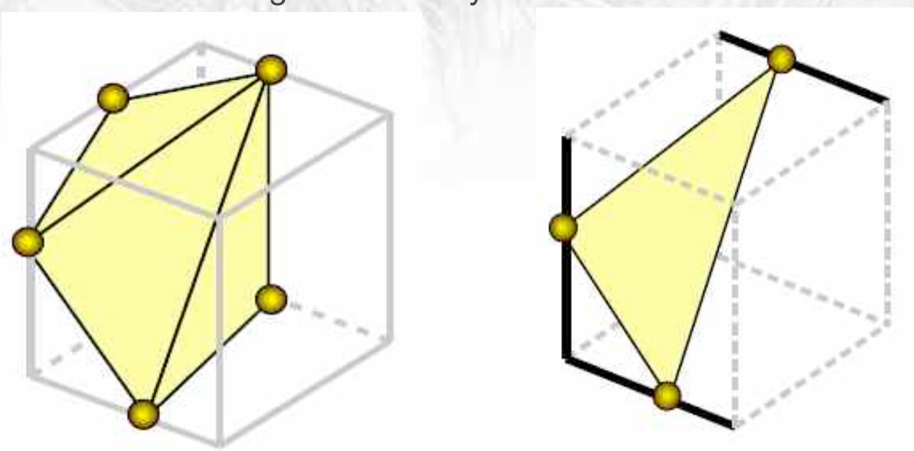

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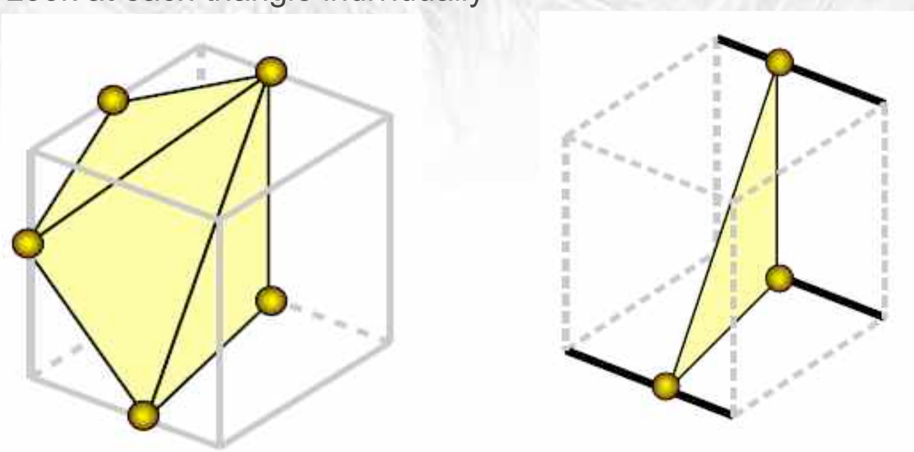

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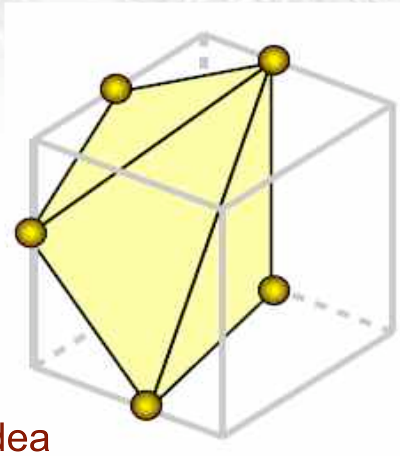 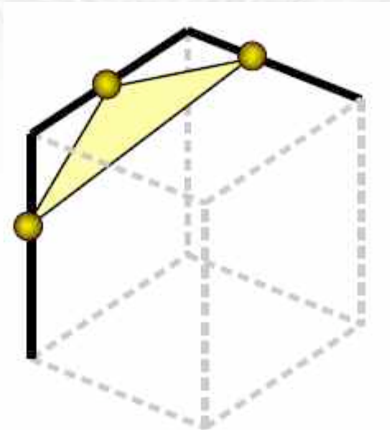

Idea
Since three active edges define a triangle, we could evaluate all possible triangles generated by such configuration

Triangle Quality of Marching Cubes

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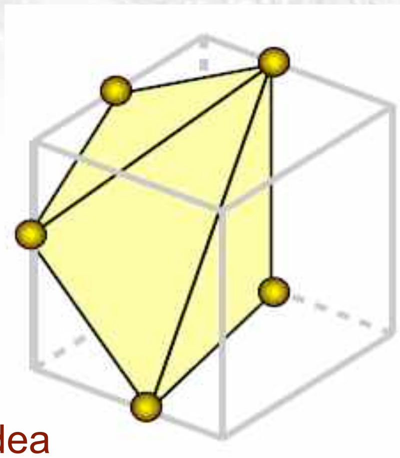 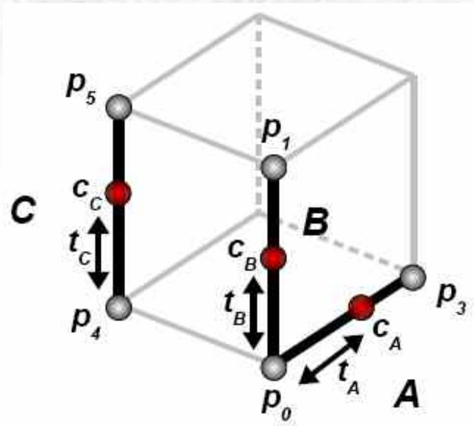

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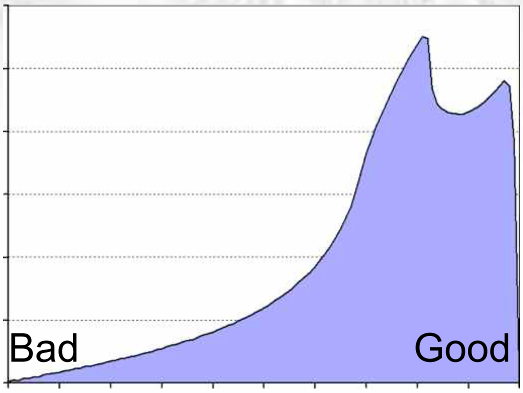
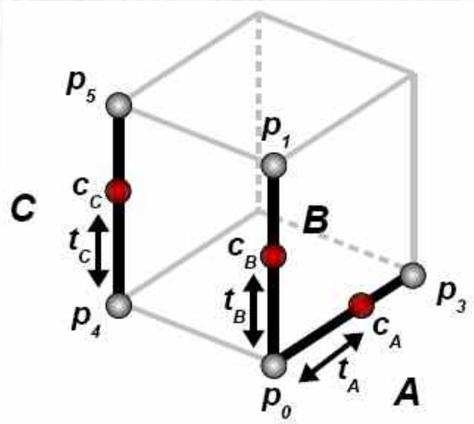


histogram of triangle quality



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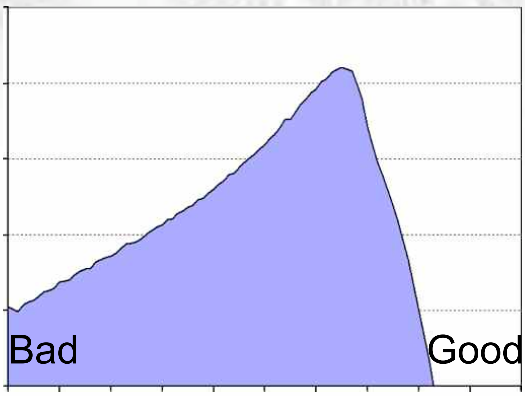
Triangle Quality of Marching Cubes

- Several conclusions can be derived



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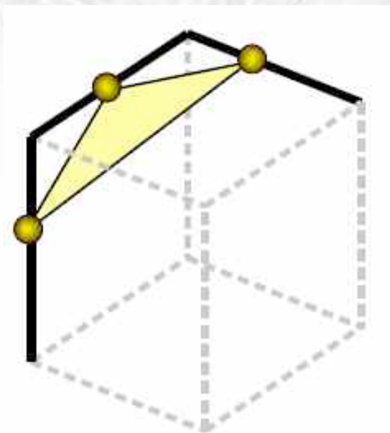

Interesting Observation

This configuration is responsible for the worst triangles of MC !!!

Triangle Quality of Marching Cubes

- Other Questions to answer:
  - How many cases of edge configurations ?

Triangle Quality of Marching Cubes

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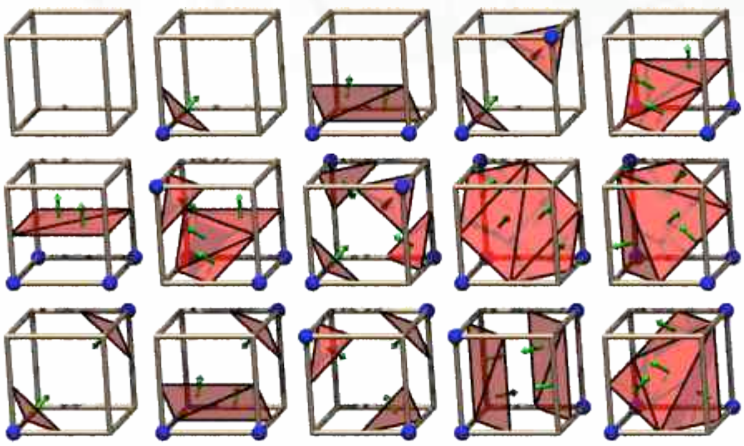
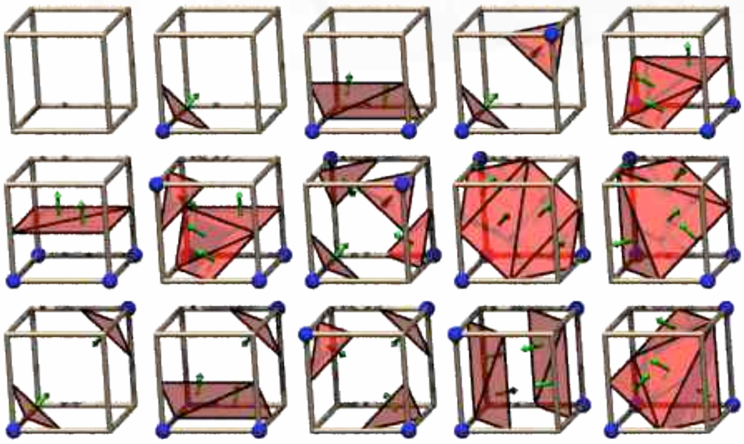
  - Is it similar to Marching Cubes Cases Enumeration ?



Triangle Quality of Marching Cubes

- Other Questions to answer:
  - How many cases of edge configurations ?
  - Is it similar to Marching Cubes Cases Enumeration ? YES, but at edge level



Outline

- Motivation
- Edge Groups in Cubic Cells
- Improving Triangle Quality Based on Edge Groups
- Results
- Conclusions

Edge Groups in Cubic Cells

- Counting Cases in MC
  - [Lorensen and Cline 87] 15 cases based on inspection
  - [Nielson et al 2002] 23 cases by rotation only
  - [Banks et al 2004] formal proof using group theory

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- Counting Cases in Edge Configurations:
  - Edge configuration (e_0, e_1, e_2): 3 different edges of the cube
  - Edge (Symmetry) Group: collection of edge configurations that can be transformed into another by symmetry transformations (e.g. rotations and improper reflections)

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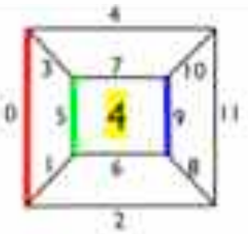
  - Edge (Symmetry) Group: collection of edge configurations that can be transformed into another by symmetry transformations (e.g. rotations and improper reflections)

(0,5,9)


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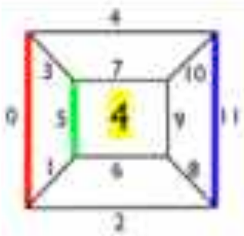

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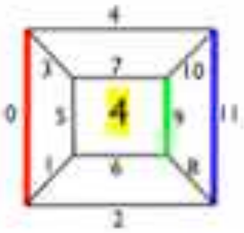
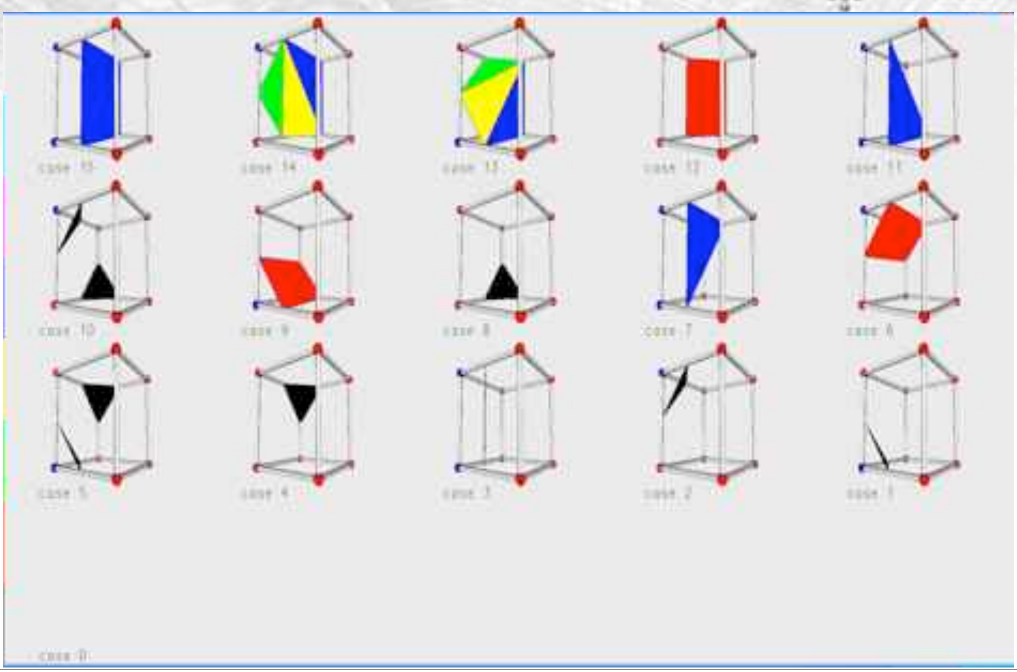


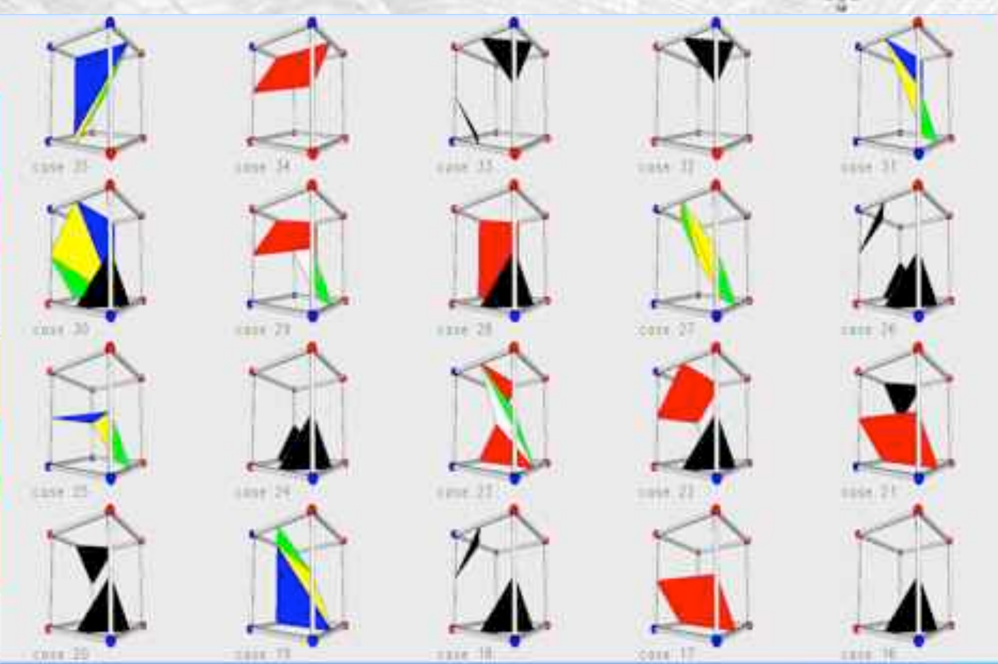
Counting Cases in Edge Groups

- True to the original Marching Cubes paper, we first did it by inspection too
- How ?

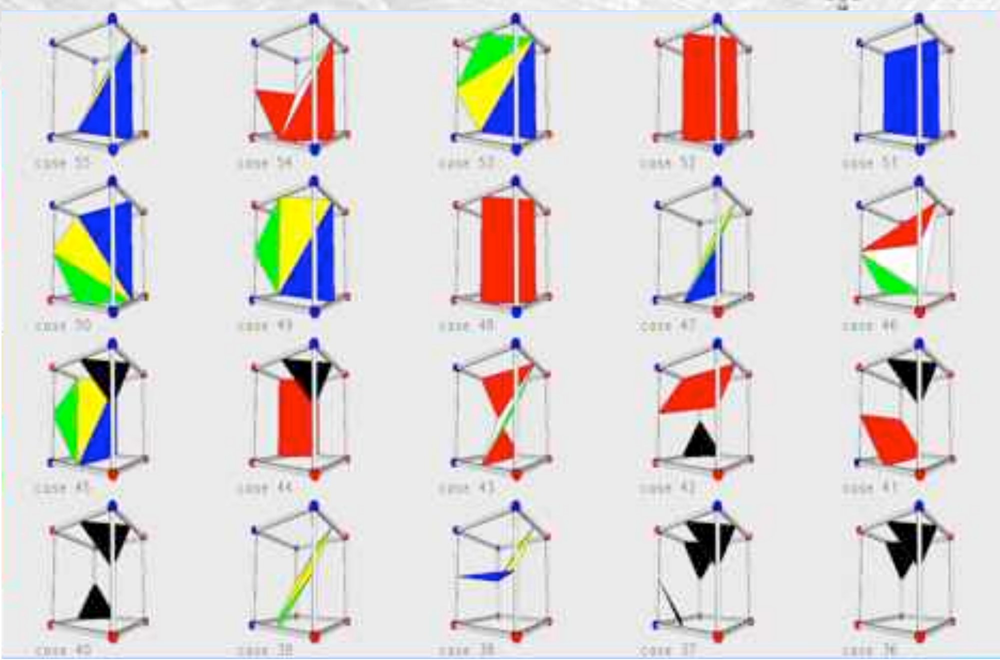
Edge Groups in the MC table



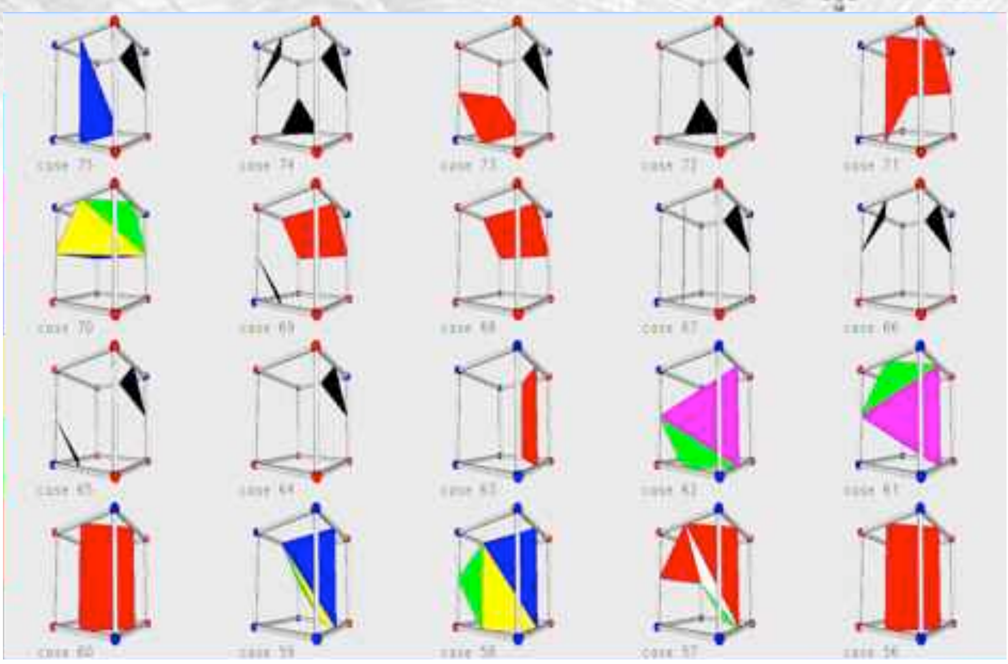
Edge Groups in the MC table



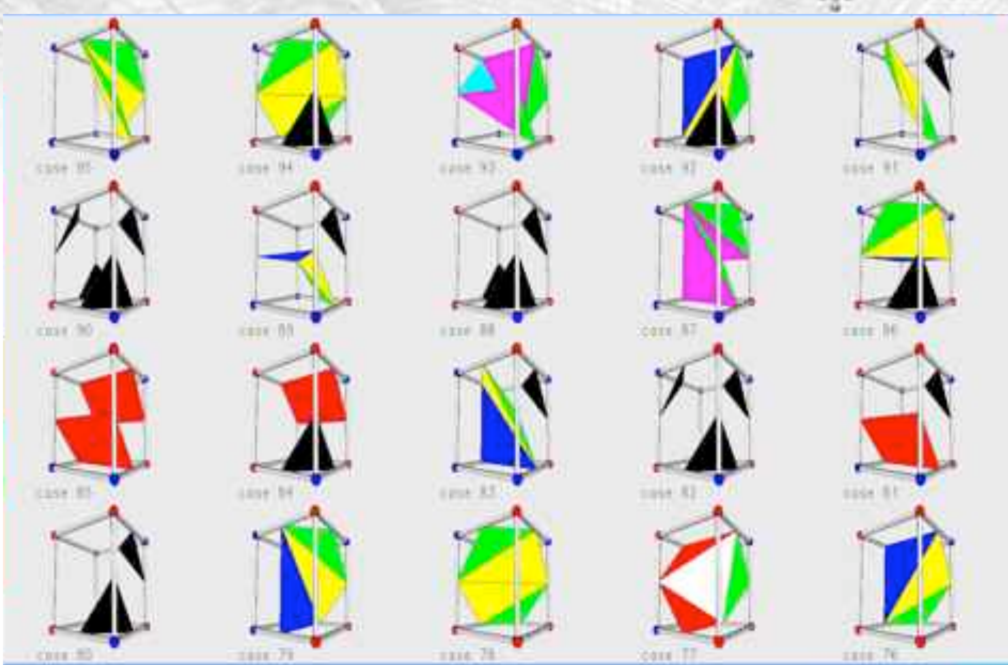
Edge Groups in the MC table



Edge Groups in the MC table



Edge Groups in the MC table



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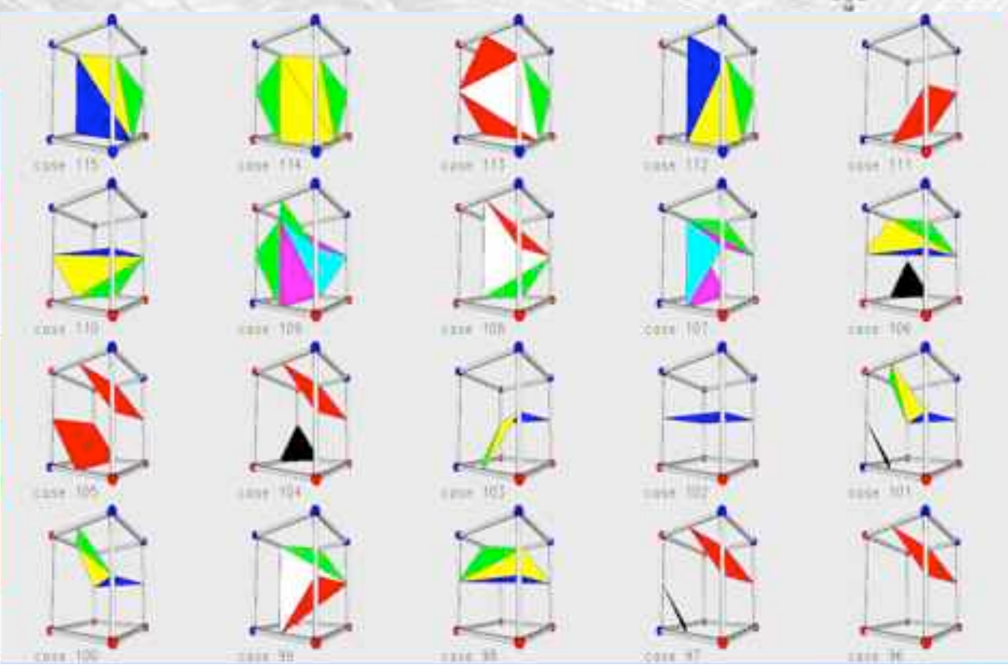
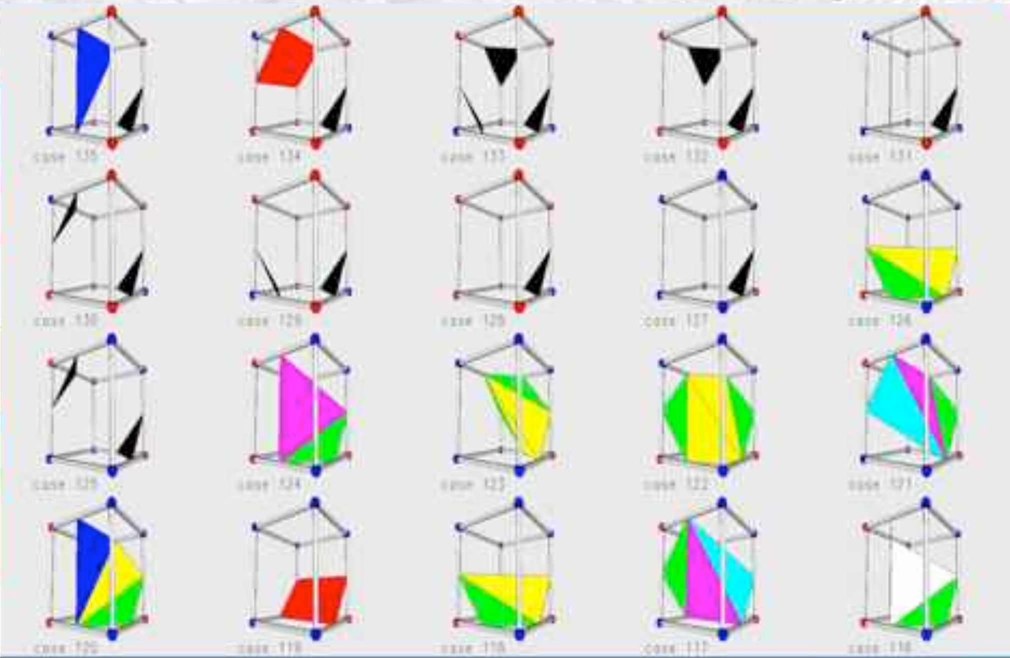
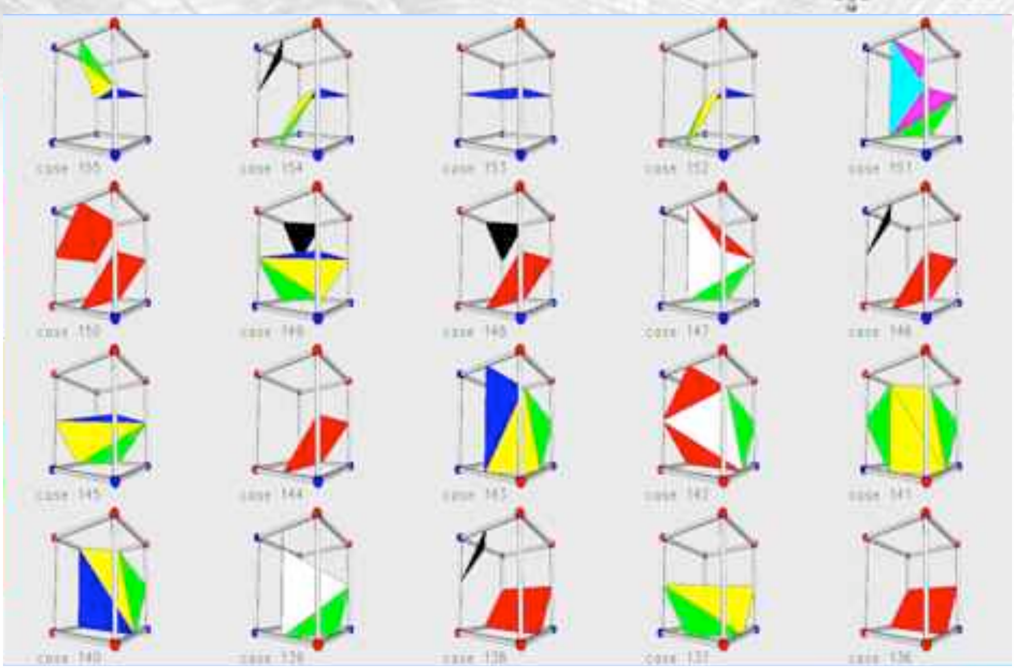


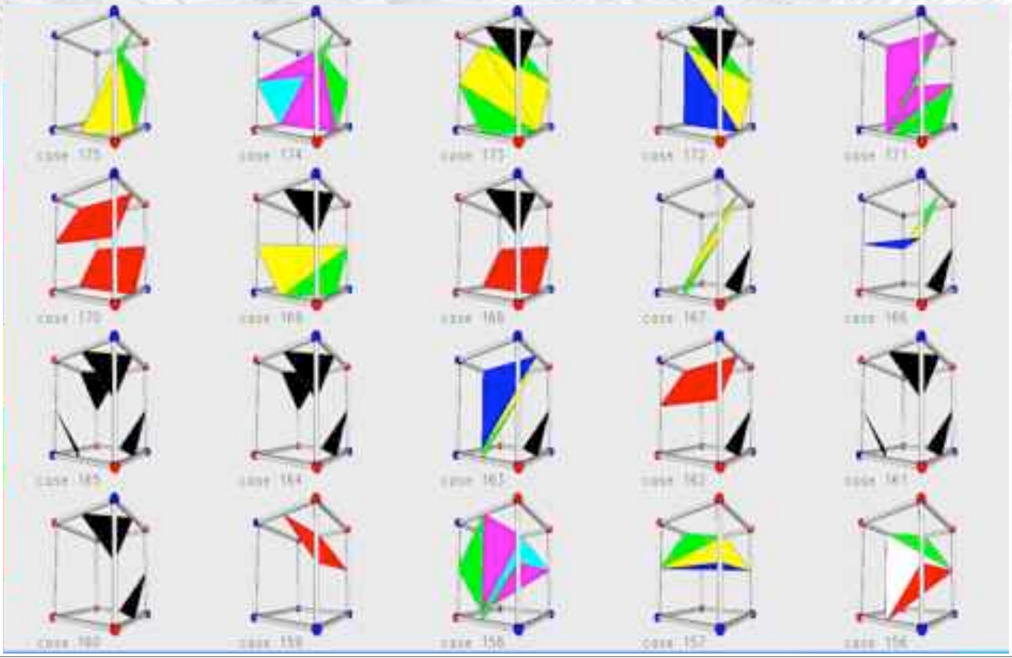
Edge Groups in the MC table



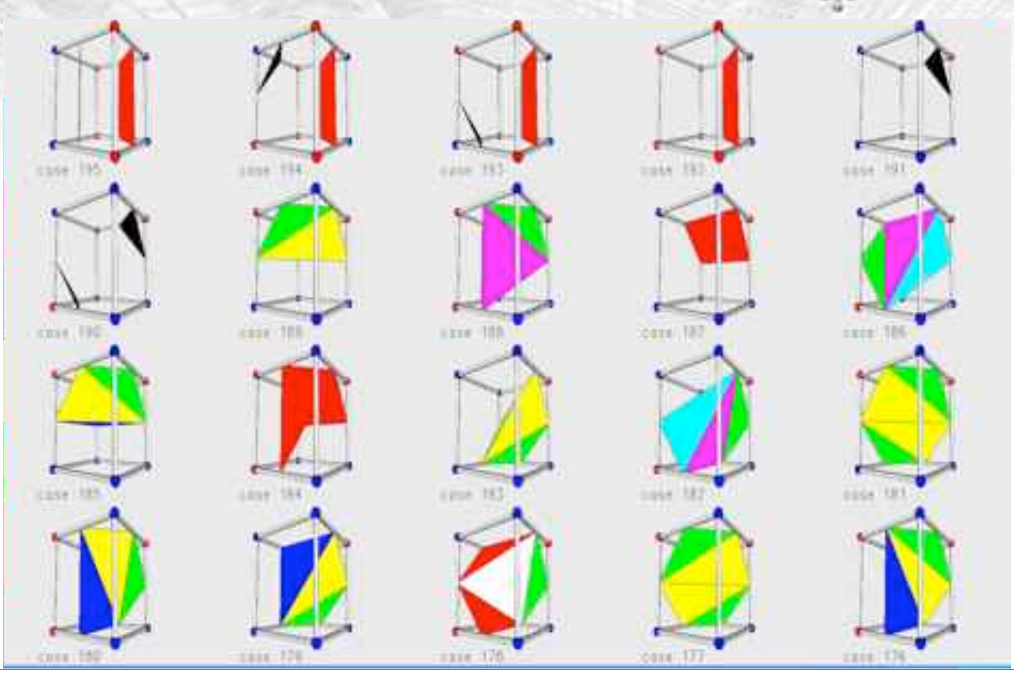
Edge Groups in the MC table



Edge Groups in the MC table



Edge Groups in the MC table



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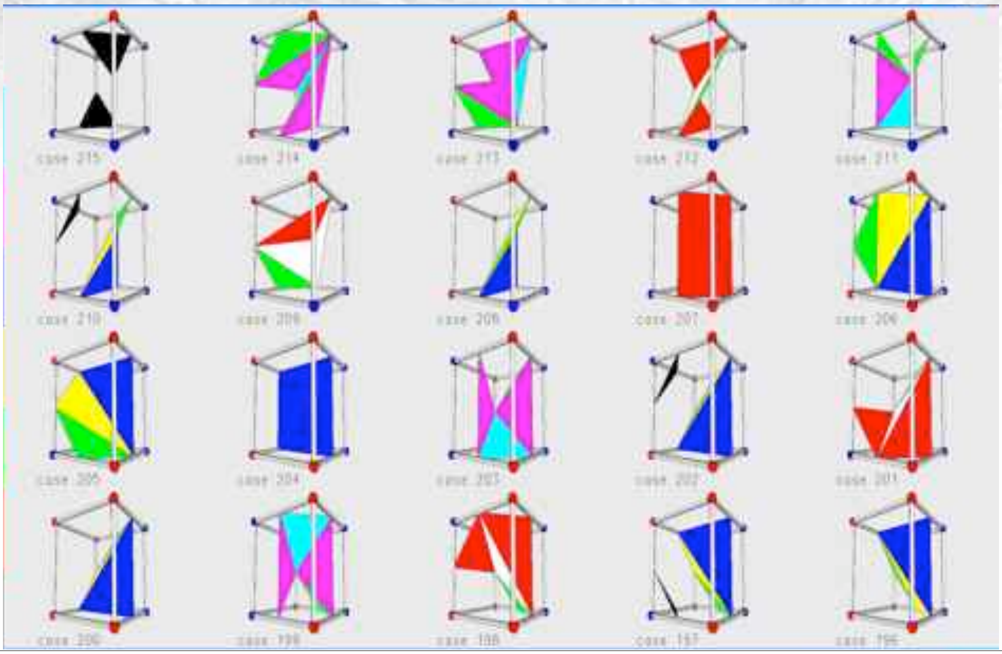

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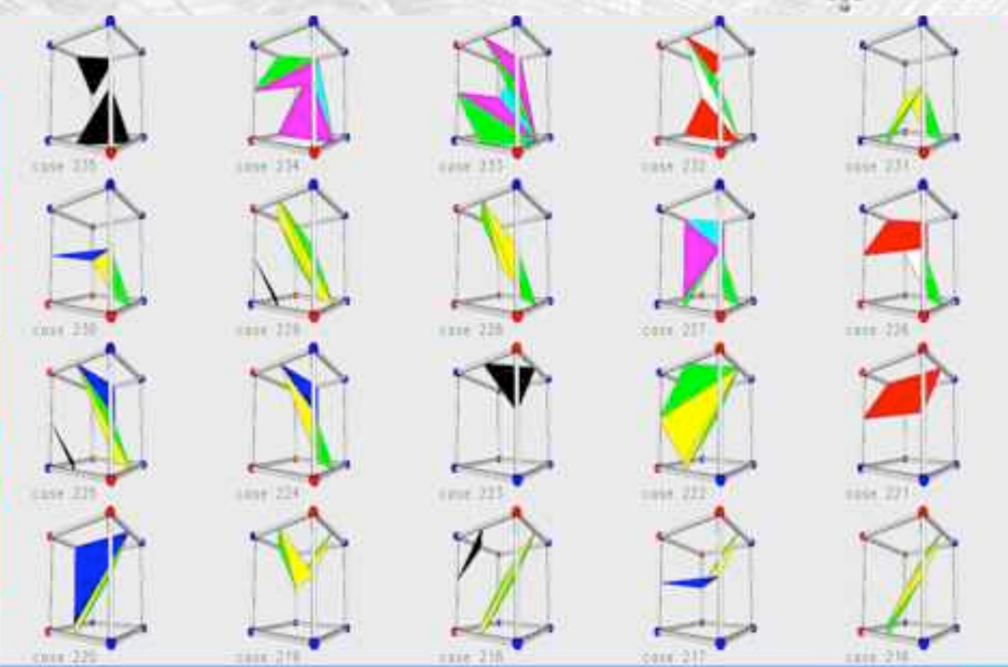
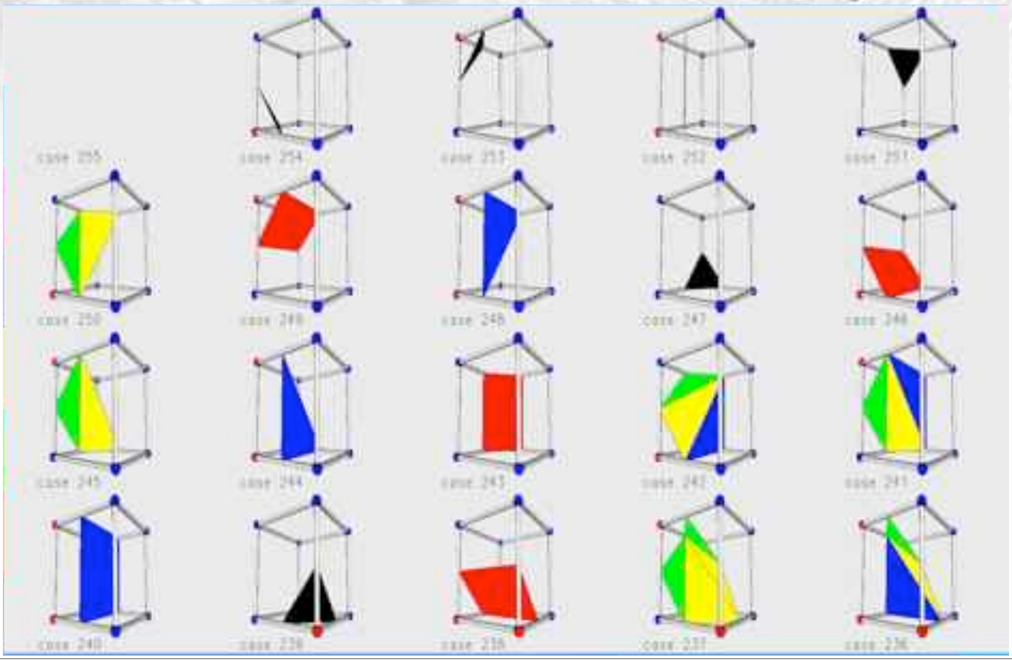
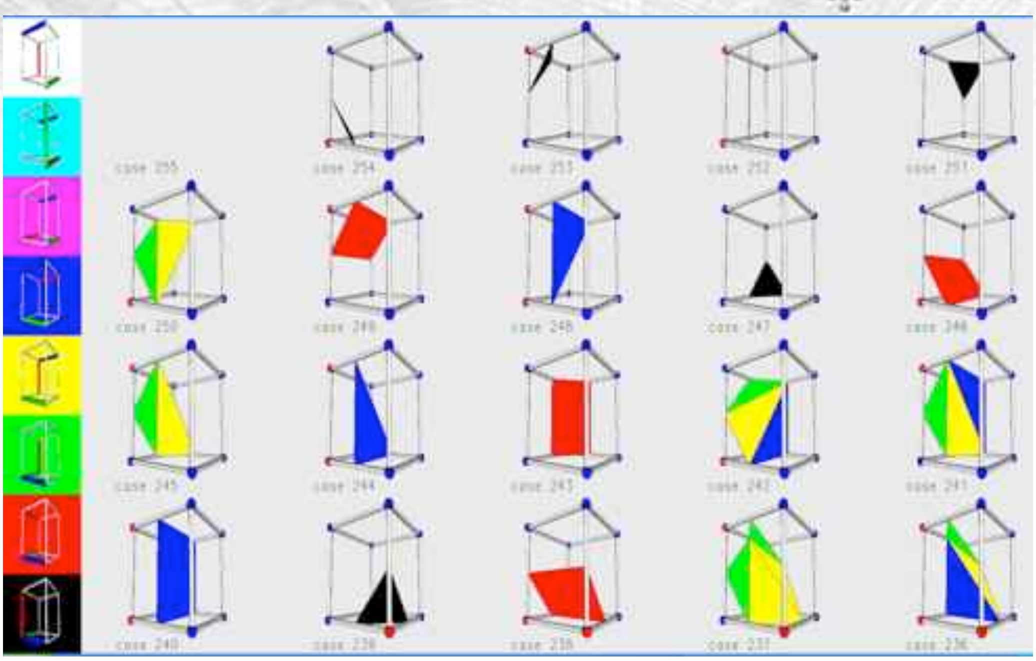


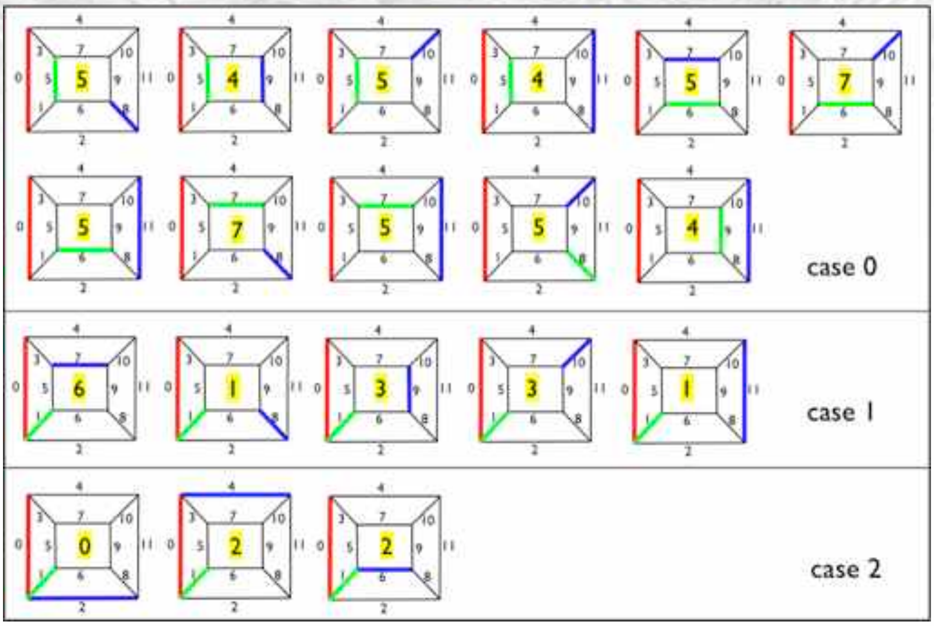
Edge Groups in the MC table



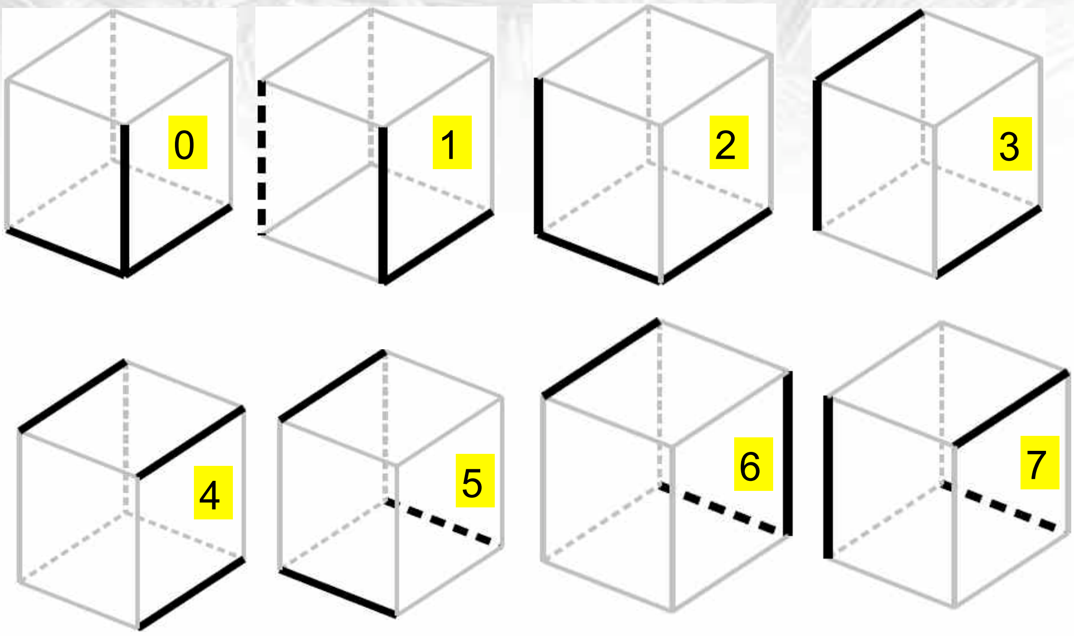
Edge Groups in the MC table



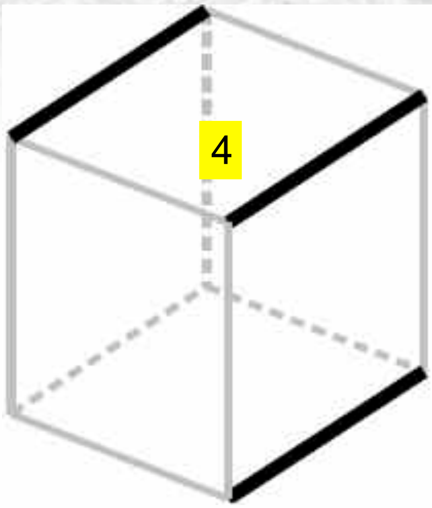
Edge Groups proof (paper)

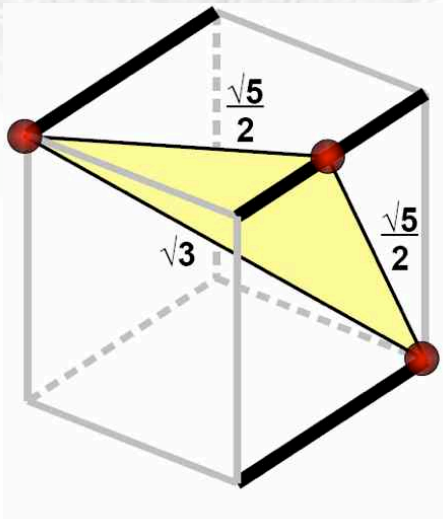


8 Edge Groups in Cubic Cells

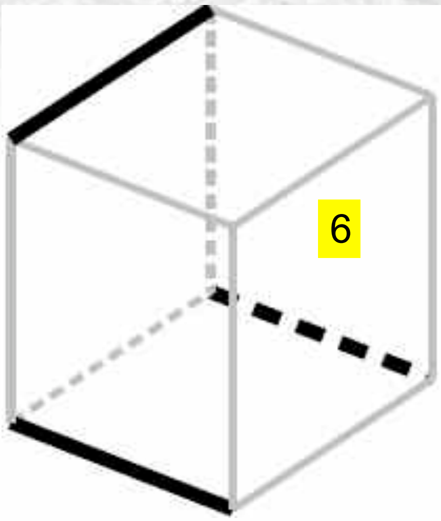


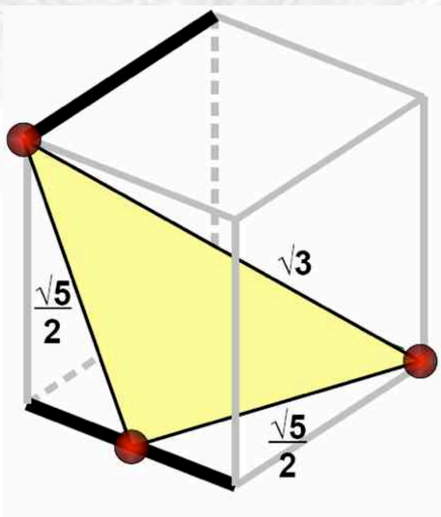
Triangle quality analysis: Worst triangle of case 4



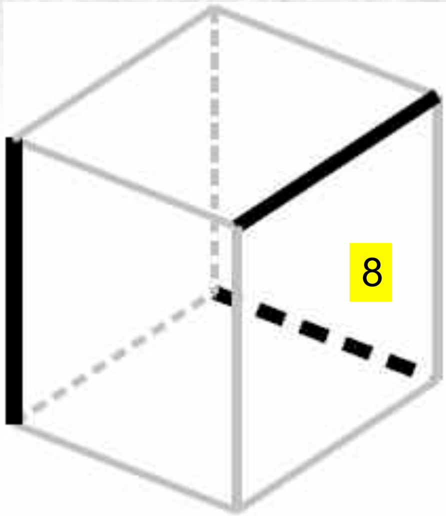


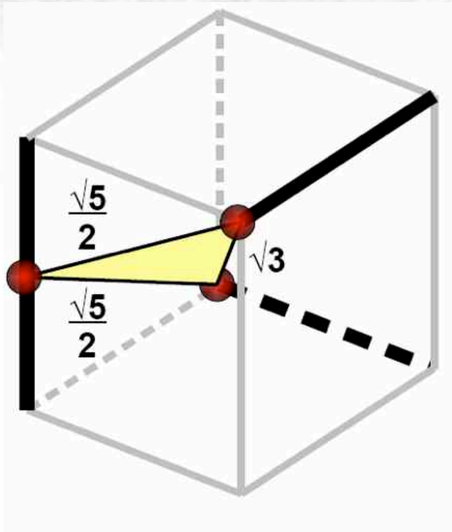
Triangle quality analysis: Worst triangle of case 6



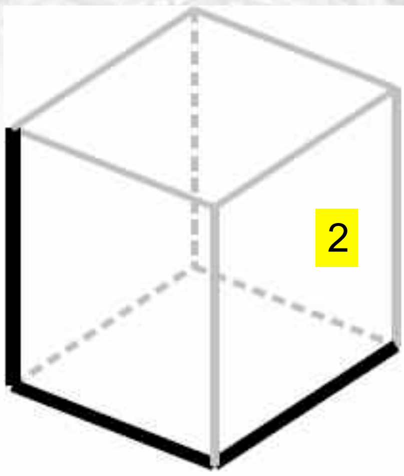


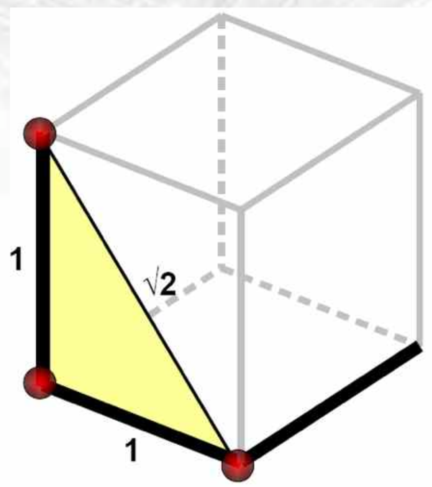
Triangle quality analysis: Worst triangle of case 8



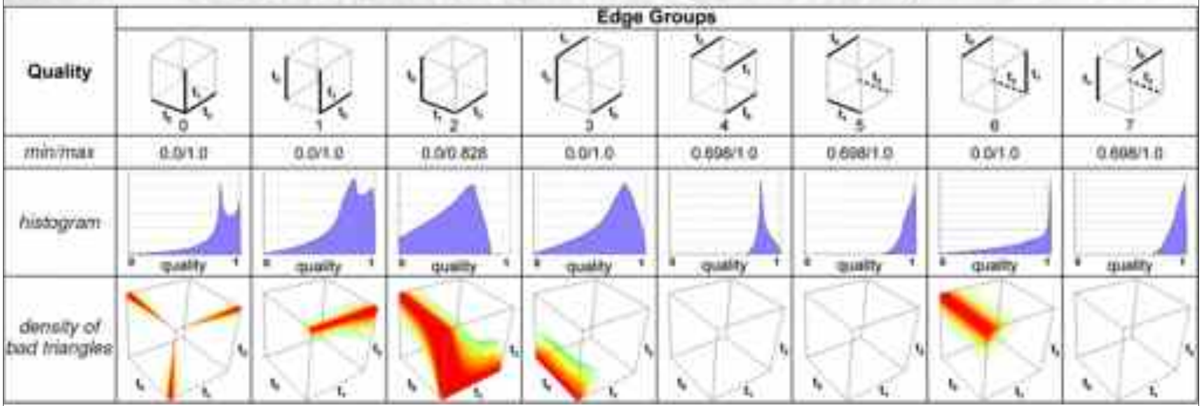


Triangle quality analysis: Best triangle of case 2

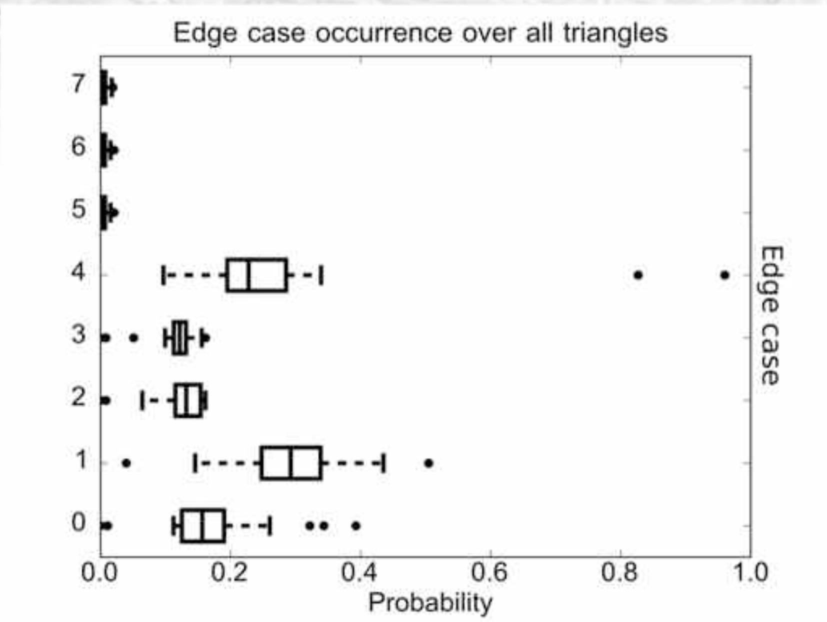




Triangle Quality Analysis: Edge Groups

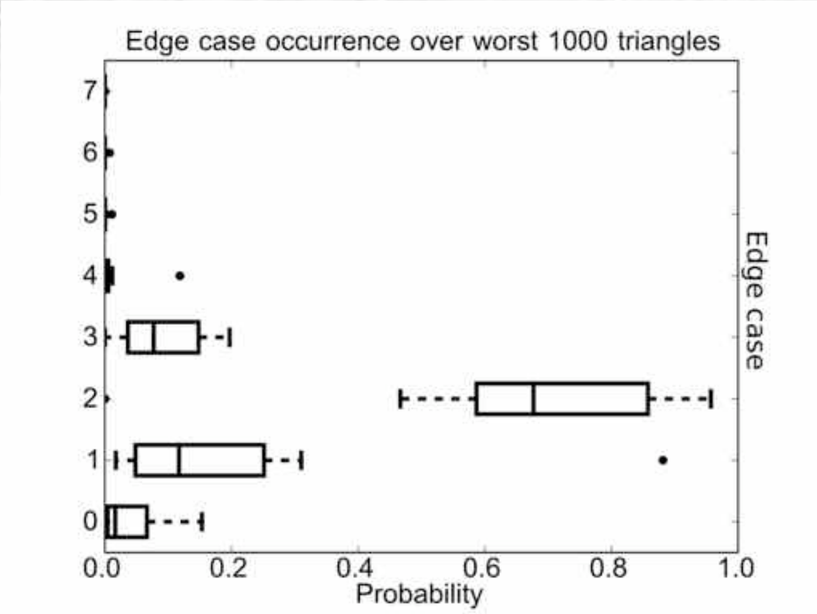


Edge Groups Occurrence



30 datasets

Interesting Observation: Edge Group 2 dominates



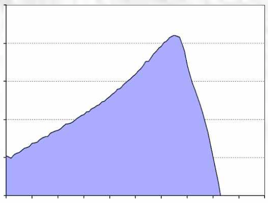
30 datasets

Outline

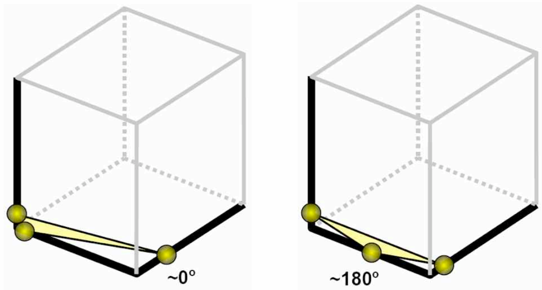
- Motivation
- Edge Groups in Cubic Cells
- Improving Triangle Quality Based on Edge Groups
- Results
- Conclusions

Improving Triangle Quality based on Edge Groups

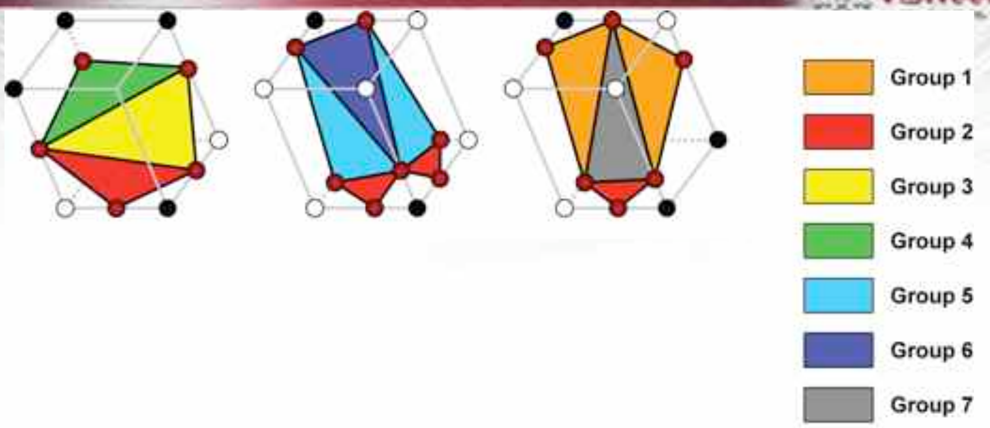
- Observation 1 : Edge Group 2 generates the worst triangles
  - Changing the MC table

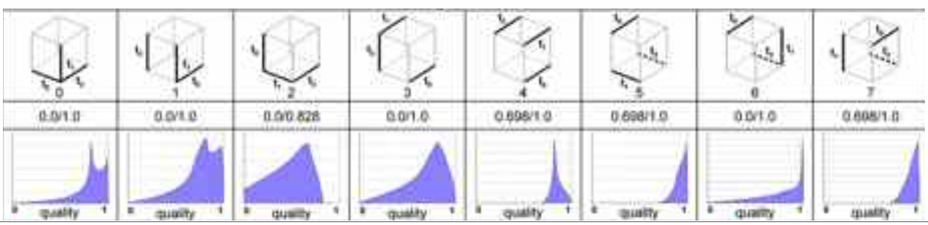


- Observation 2: Bad triangles when isosurface is parallel to the edge
  - Edge transformations using Macet [Dietrich et al 2009]

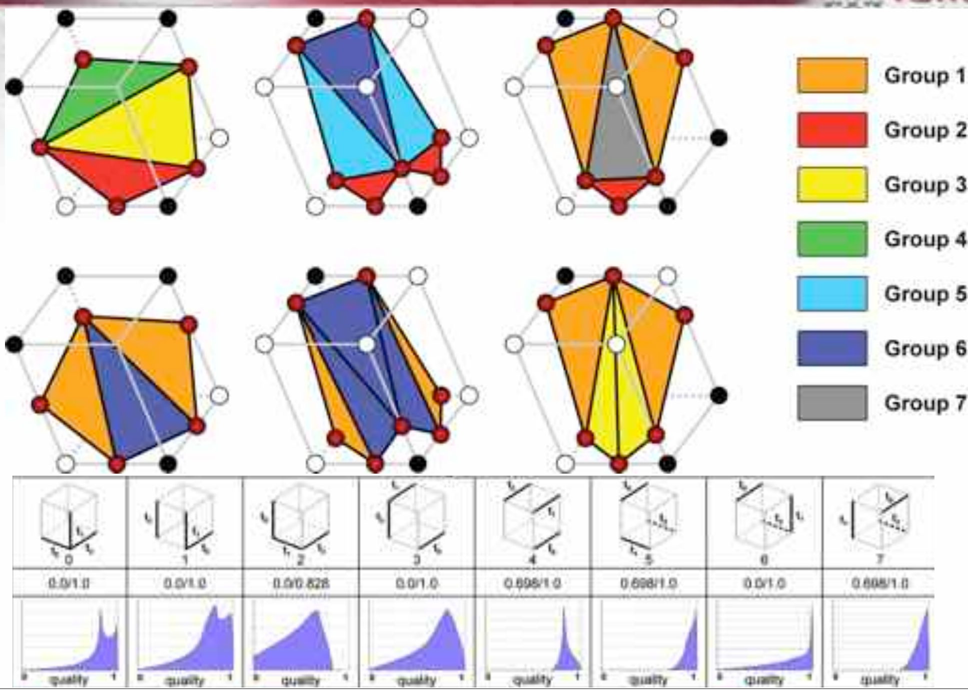


Improvement 1: Changing MC Table



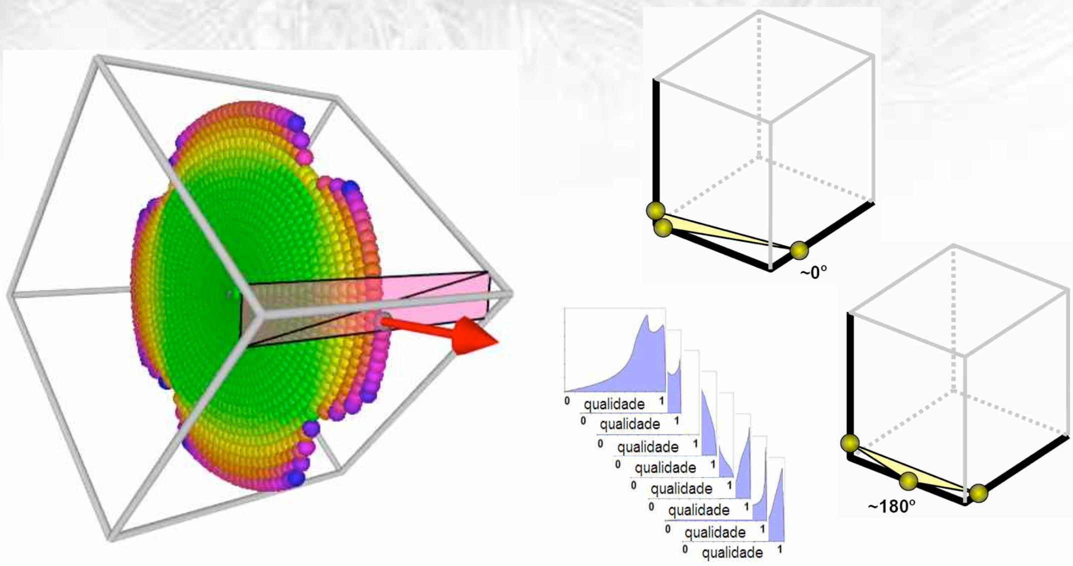


Improvement 1: Changing MC Table



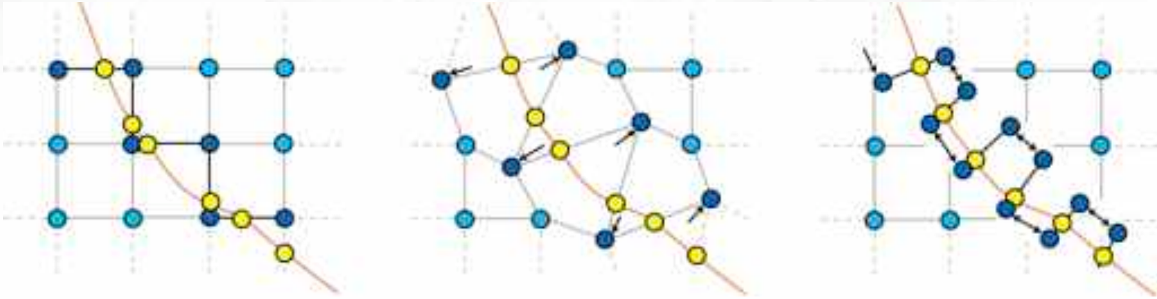
Evaluating Situations where bad triangles arise

- Bad cases happen when the isosurface is parallel to the edge



Improvement 2: Edge Transformations using Macet

- Macet [Dietrich et al 2009]
  - Edge transformations alter positions of grid points in MC

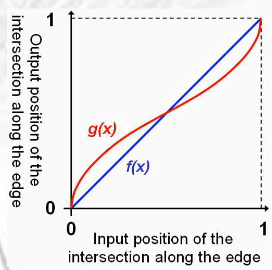


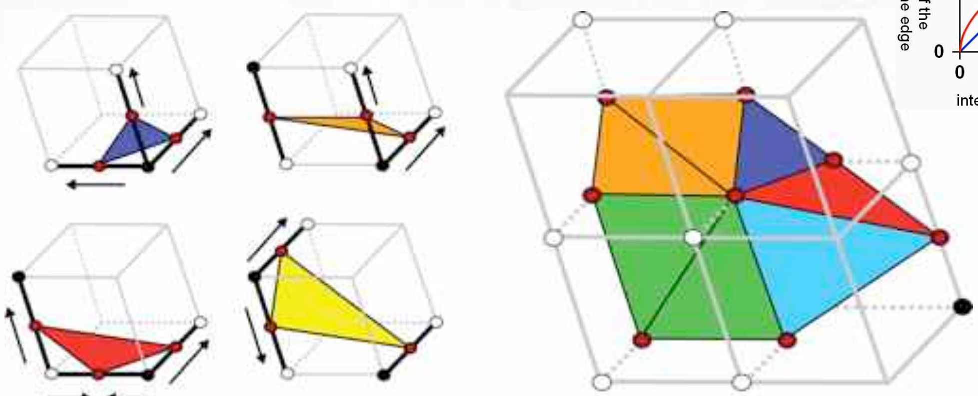
Gradient Transformation

Tangent Transformation

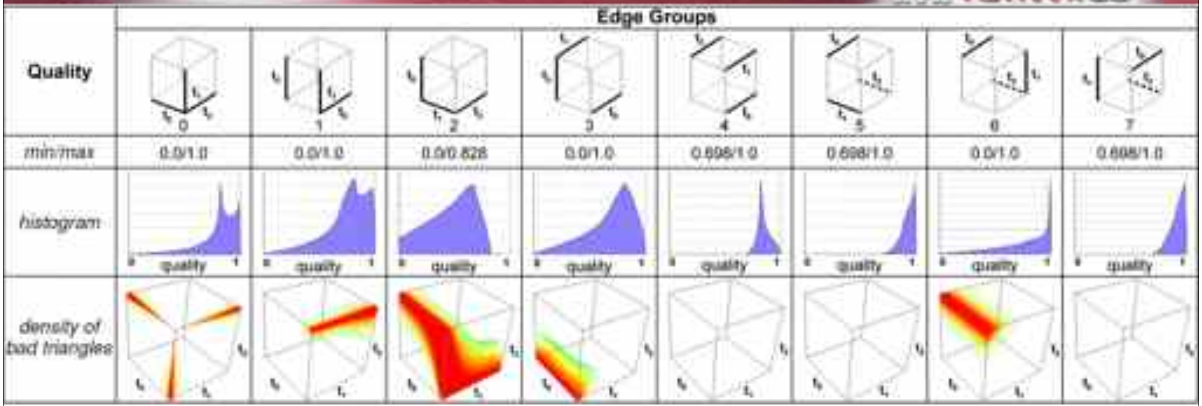
Improvement 2: Edge Transformations using Macet

- Displacement of Intersection Vertices

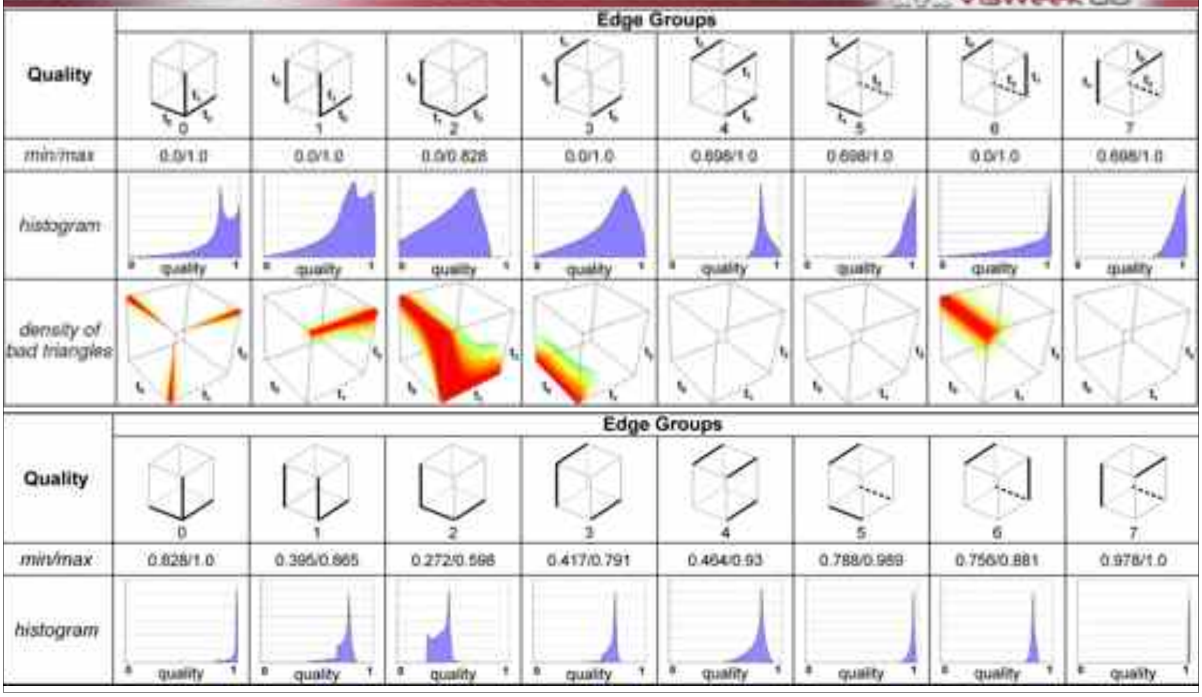




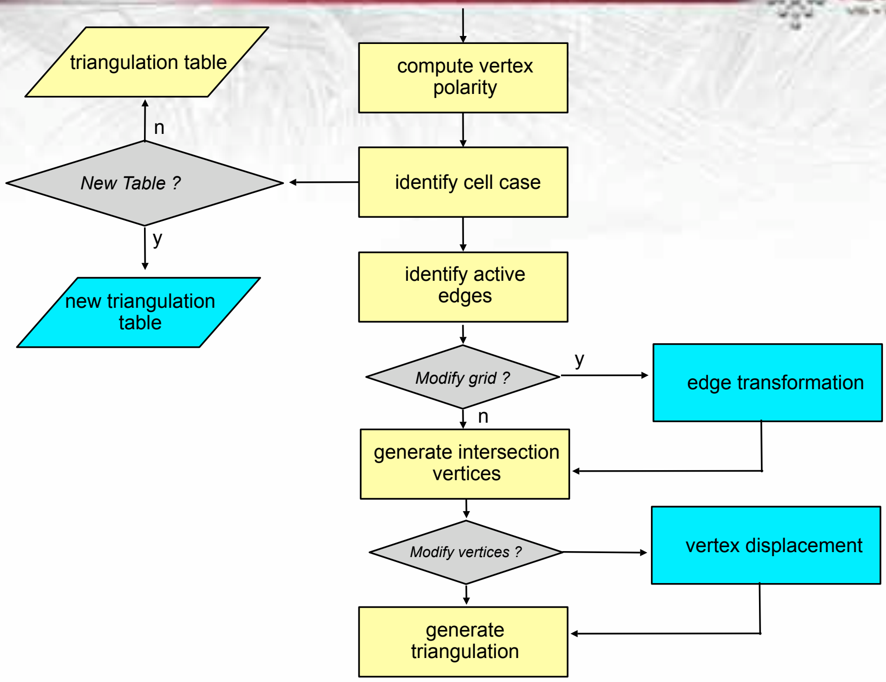
Improvement 2: Edge Transformations using Macet



Improvement 2: Edge Transformations using Macet



Summary of Changes to Marching Cubes Code



Outline

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Results

	Original MC Old Table	Macet Old Table	Original MC New Table	Macet New Table Displacements
Silicium @ 140.5	0.00157			
Engine @ 49.5	0.00042			
Bonsai @ 49.5	0.00013			
Lobster @ 49.5	0.00088			

Results

	Original MC Old Table	Macet Old Table	Original MC New Table	Macet New Table Displacements
Silicium @ 140.5	0.00157		0.01792	
Engine @ 49.5	0.00042		0.00080	
Bonsai @ 49.5	0.00013		0.00152	
Lobster @ 49.5	0.00088		0.00153	

Results

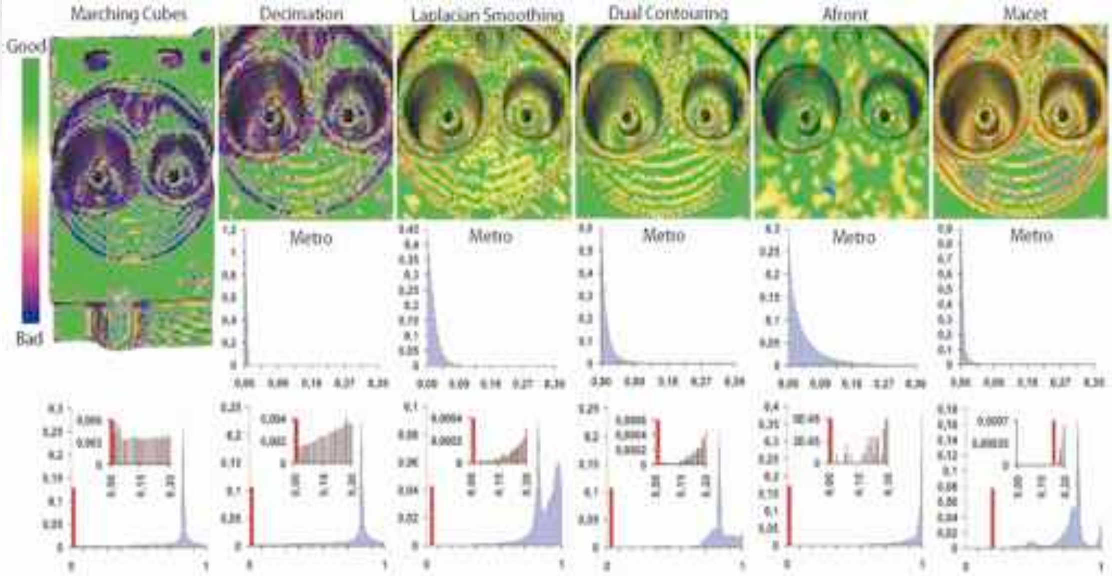
	Original MC Old Table	Macet Old Table	Original MC New Table	Macet New Table Displacements
Silicium @ 140.5	0.00157	0.199	0.01792	
Engine @ 49.5	0.00042	0.235	0.00080	
Bonsai @ 49.5	0.00013	0.102	0.00152	
Lobster @ 49.5	0.00088	0.116	0.00153	

Results

	Original MC Old Table	Macet Old Table	Original MC New Table	Macet New Table Displacements
Silicium @ 140.5	0.00157	0.199	0.01792	0.344
Engine @ 49.5	0.00042	0.235	0.00080	0.293
Bonsai @ 49.5	0.00013	0.102	0.00152	0.195
Lobster @ 49.5	0.00088	0.116	0.00153	0.247

Results (Previous Version of Macet)

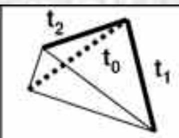
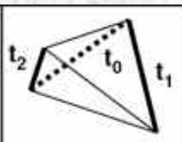
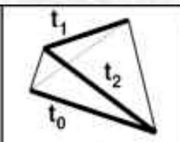
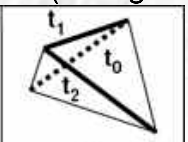
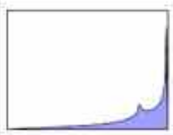
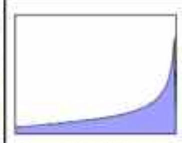
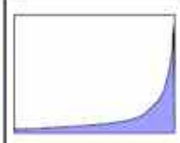
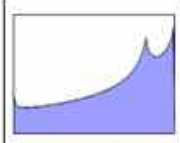
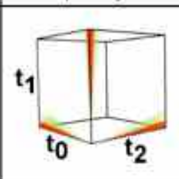
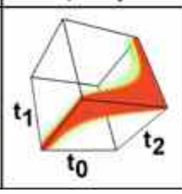
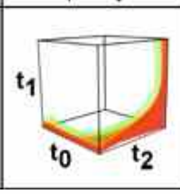
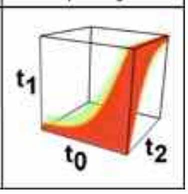
- Comparison Against Other Methods



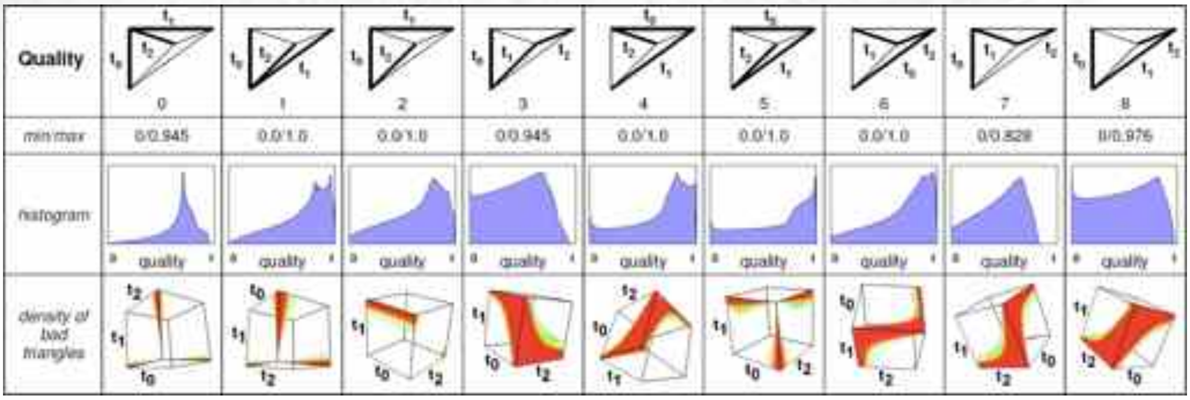
Edge Groups in Tetrahedra Cells

- Extensions to tetrahedra cells

BCC Tetrahedra (4 Edge Groups)

Quality	 0	 1	 2	 3
min/max	0.0/1.0	0.0/1.0	0.0/1.0	0.0/1.0
histogram	 0 quality 1	 0 quality 1	 0 quality 1	 0 quality 1
density of bad triangles				

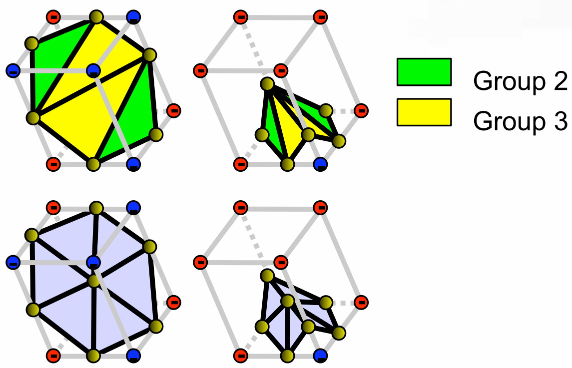
Edge Groups in Tetrahedra Cells

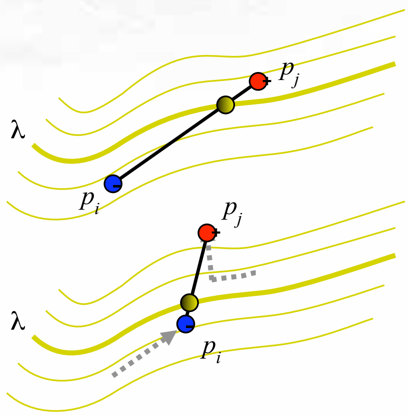


Kuhn Tetrahedra (9 Edge Groups)

Ongoing Work

- Unified Edge Transformations Macet (Technical Report coming soon)
  - Single Edge Transformation
  - Optional Vertex Displacement
  - More efficient, more quality





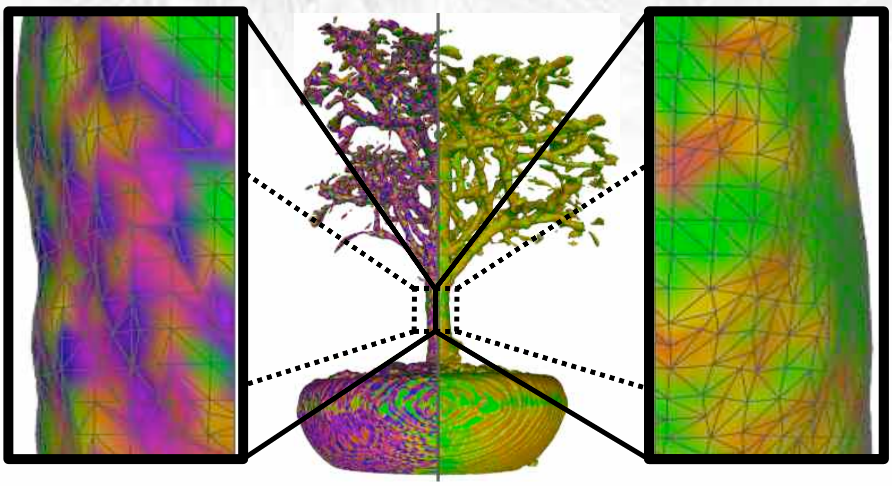
Latest Quality and Performance Results

- Unified Edge Transformations Macet

	MC		Unified Macet	
	time	*quality*	*time*	*quality*
Bonsai	1.4	0.0002	9.9	0.32
Engine	1.1	0.0002	8.6	0.35
Hydrogen	0.15	0.069	0.7	0.52
Laçador	0.7	4.8e-7	2	0.37
Neghip	0.07	0.0031	0.64	0.38
Siliicium	0.03	0.0009	0.38	0.41

Demo

- Side-by-side comparison: MC x Unified Macet



Marching Cubes

Macet

Conclusions

- Main contributions:
  - Edge groups proposal and analysis in MC:
    - Histogram analysis of each individual case
    - Case 2 generates the worst triangles
  - Modifications in MC
    - Change of MC table
    - Improvements and Validation of Macet
  - Reproducibility (open source):
    - http://www.sci.utah.edu/~csheid/vis2008/edge-groups (Vistrails)

Acknowledegments

- CNPq (Brazil) scholarship and grants 485853/2007-0, 453538/2008-8
- IBM Ph.D. Student Fellowship.
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- National Science Foundation (grants CNS-0751152, CCF-0528201, OCE-0424602, CNS-0514485, IIS-0513692, CCF-0401498, OISE-0405402, CNS-0551724)
- IBM Faculty Awards (2005, 2006, and 2007)
- Reviewers for feedback

Questions ?